XXXIV. *On the Construction of Life-Tables, illustrated by a New Life-Table of the Healthy Districts of England.* *By* W. FARR, *Esq., M.D., F.R.S.*

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CONTENTS:—

THE Transactions of the Royal Society contain the first Life-Table. It was constructed by HALLEY, who discovered its remarkable properties, and illustrated some of its applications. The Breslau observations did not supply HALLEY with the data to frame an accurate Table, for reasons which will be immediately apparent; but the conception is full of ingenuity, and the form is one of the great inventions which adorn the annals of the Royal Society.

Tables have since been made correctly representing the vitality of certain classes of the population; and the form has been extended so as to facilitate the solution of various questions.

In deducing the English Life-Tables from the National Returns, I have had occasion to try various methods of construction; and I now propose to describe briefly the nature of the Life-Table, to lay down a simple method of construction, to describe an extension of its form, and to illustrate this by a new Table representing the vitality of the healthiest part of the population of England.

The Life-Table is an instrument of investigation; it may be called a *biometer*, for it gives the exact measure of the duration of life under given circumstances. Such a Table has to be constructed for each district and for each profession, to determine their degrees of salubrity. To multiply these constructions, then, it is necessary to lay down rules, which, while they involve a minimum amount of arithmetical labour, will yield results as correct as can be obtained in the present state of our observations.

I. GENERAL DESCRIPTION OF A LIFE-TABLE. (See Table C, p. 870.)

A Life-Table represents a *generation of men* passing through *time*; and time under this aspect, dating from birth, is called age. In the first column of a Life-Table *age* is expressed in *years*, commencing at 0 (birth), and proceeding to 100 or 110 years, the extreme limit of observed life-time.

If we could trace a given number of children, say 100,000, from the date of birth, and write the numbers down that die in the first year, living therefore less than one year, against 0 in the Table, and on succeeding lines the numbers that die in the second, third, and every subsequent year of age until the whole generation had passed away, these numbers would form a *Table of Mortality*, showing at what ages 100,000 lives become extinct.

Again, if the 100,000 children were followed, and the numbers living on the first, on the second, and on every subsequent birthday until none was left, the column of numbers would constitute a *Table of Survivorship*. So if of 100,000 children born at a given point of time, the numbers dying (d_x) in each subsequent year were written in one column, and the numbers surviving (l_x) at the end of each year in another column, the two primary columns of the Life-Table would be formed.

It is evident that if one of these columns is known the other may be immediately deduced from it; for if of 100,000 children born 10,295 die in the first year of age, 3005 in the second year of age, it follows that the numbers living at the end of one year must be 89,705, at the end of two years 86,700. Upon adding the column (d_x) from the bottom up to the number against any age (x), the sum will represent the whole of the numbers *dying after that age*; and consequently the numbers *living at that age*, as shown in the collateral column (l_x).

The 100,000 children born at the same moment, and counted *annually* to determine the numbers *living* at *the end of every year*, would by our Table completely pass away in less than 107 years. If another generation of 100,000, born a year afterwards, were followed, the numbers dying in the various years of age would not be very different, the circumstances remaining the same; and the numbers of those entering each year of age would vary inconsiderably from those of the first series. If 100,000 children again were born at annual intervals, and were subject to an invariable law of mortality, they would form a community of which the numbers living at each age would be represented by the sucessive numbers (l_x) in the Life-Table. The sum of these numbers, by the new Table of Healthy Districts, would be 4,951,908. The births are here assumed to take place simultaneously at annual intervals; immediately before the births, therefore, in such a community its population would be 4,851,908, to which it would fall progressively from 4,951,908 by 100,000 successive deaths in the year. The average number constantly living would be some number between 4,951,908 and 4,851,908; and it would be very nearly the mean of these limiting numbers.

In the ordinary course of nature, the births in a community take place in remittent succession; and if it is assumed that the 100,000 births occur at equal intervals over every year, it is evident that at any given date a certain number will be found living at all the intermediate points of age between 0 to 1 year, 1 to 2, 2 to 3, and all the remaining years of age. The population in the above instance would be found by enumeration to be nearly 4,899,665.

The annual *births* would be 100,000 in such a community. The annual deaths would also be 100,000; and by taking out the deaths at each year of age, from the parish registers of a single year, the second column (d_x) of the Life-Table would be found. By adding this column of deaths up and entering the sum of the numbers year by year against every year of age (x), the third column (l_x) of the Life-Table would be obtained; for it has been already shown that the numbers attaining any age x are equal to the numbers dying at that age, and all the subsequent ages. From the registers of the deaths, a Table of the numbers of the *population living* in a parish *so constituted* could be immediately determined without any enumeration. Its deviations from the truth would be accidental; and they would be set right by taking the mean of many years. So also from a simultaneous enumeration of the *numbers living in each year of age*, the two columns d_x and l_x of the life could be constructed without reference to any registry of the deaths at different ages.

The *mean age at death* in such a community would express the mean lifetime, or the expectation of life at birth; and the product of the number expressing the annual births multiplied into the mean age at death would give the numbers of the population.

The facts which a Life-Table expresses in numbers may be represented by the lines of a figure; age (x) being indicated by the abscissas measured from 0, the *numbers living* (l) at each age by the ordinates of a curve line, and the numbers living between any two ages by the plane surface within the two ordinates, the curve line, and the corresponding portion of the abscissa. The relative numbers living at the ages 20 and 21 are seen in the two lines of Plate XLII. fig. 1, over the ages 20 and 21; if the deaths in the intervening year all occurred immediately after the age 20 was attained, the numbers living would also be represented by the parallelogram having its two sides equal to the ordinate over 21, and for its base the portion of the abscissa between 20 and 21; but if all the deaths occurred only the instant before the age 21 was attained, the height of the parallelogram would be represented by the ordinate over the age of 20. The deaths occur at intervals between the two ages, so the numbers living, and the *lifetime* which is passed between the two ages, are correctly represented by the curvilinear area.

The deaths in each year of age are called the *decrements of life*. They are represented by the differences in the lengths of the successive ordinates. Thus by cutting off a small portion of the ordinate at the age 20, the ordinate at the age 21 is obtained; this small portion, shown in Plate XLII., represents the decrement of life in that year of age. It will be observed that the decrements vary at every year of age; and this is more evident when they are exhibited on the larger scale of Plate XLII. fig. 2. The decrement in the first year is large; in the first five years the decrements of life are considerable; at the age of 10 to 15 they fall to their minimum; slowly increase to the age of 56; increase more rapidly until the maximum is attained at the age of 75; then decline gradually to 85, and after that more rapidly until every life is extinct at the age 107 by this Table.

II. PRINCIPLES OF CONSTRUCTION. THE FUNDAMENTAL COLUMN l_x.

The conditions of the hypothesis upon which the preceding reasoning rests are never precisely realized in nature; in the first place the number of births fluctuates, increases, or decreases from year to year, and the deaths fluctuate still more; rarely equalling the births in number. Immigration and emigration interfere. Under these circumstances, Tables such as those which HALLEY, PRICE and others made from the observations on the *deaths alone* are never accurate, and require correction to give approximate results. If it be assumed that the law of mortality remains invariable, and that migration does not interfere, then the nature of the correction to be applied to a Table framed from the deaths alone will become immediately apparent by an example. The births increase in England. Let the annual births in a portion of the community be doubled in sixty years, thus be 50,000 in 1796, and 100,000 in 1856; then the deaths of persons of the age of 60 in 1856 must be doubled to obtain the deaths which would have happened at that age if the annual births sixty years before these deaths had been 100,000. If the births have been accurately registered, formulæ for correcting the ordinary Table drawn up from the deaths at different ages will be suggested by the above considerations.

I now proceed to describe another method which has been adopted in framing the Table C, and is applicable wherever (1) the number of annual births, (2) the numbers of the population living at definite periods of age, (3) the deaths at the corresponding ages during a certain number of years, in any community are ascertained by observation. This method is not open to the previous objections.

The aim is to obtain equations which will describe the curve lines (Plate XLII. fig. 1) of the Life-Table, in the most direct way; and these equations may be deduced from the determined rate of mortality at certain intervals of age.

The relative numbers living at two ages, 20 and 21, can evidently be found from an equation which expresses the relation of the average numbers living and dying between those ages during a given time. This can be determined very nearly; for although the ages of the living are not ascertained with exact precision at the census, still by taking all the numbers living at the ages 15, 16, 17 years up to 24 and under 25, together, the aggregate represents very nearly the numbers living in that decenniad of life. The deaths at the same ages are obtained with at least equal accuracy from the registers of deaths. By this process, and by extending the observations over five or more years, a number of facts is obtained sufficiently great to yield average results; and it may be assumed that the ratio of the living at the ages 15—25 * to the dying in a year at the same ages 15—25 represents the annual rate of mortality at the exact age 20. So also the mortality rate at the ages 30, 40, 50 and other ages may be determined. As observations grow more exact, and the facts are multiplied, the intervals of age may be diminished to 5 years, and ultimately to 1 year.

In determining the *rate* of *mortality*, a given number of persons living a year is considered equivalent to twice that number living half a year, or to half the number living two years.

Thus if nd represent the deaths in n years out of a number amounting on an average to P during the same years, then $\frac{nd}{nP}=m=$ the rate of mortality, or the proportions of death in a *year* (always taken as the unit of time) out of *one year* of *lifetime*. It is found from all the observations hitherto made on a large scale, that the rate of mortality varies at every interval of age; but at the same age it may for the present purpose be considered invariable under similar circumstances.

m_x therefore varies in every moment of age; but I have employed it to express the mean annual rate of mortality during the year following the year of age x, $\therefore \frac{d_x}{P_x}=m_x$, where d_x indicates the deaths, P_x the year of lifetime, after the year of age x. The m_x is the expression of the force of the causes that induce death, of the death-force, *vis mortalis*; and its reciprocal $\frac{1}{m_x}=u_x$ measures the forces that sustain life, the *vis vitalis*.

The vital force under natural circumstances may by one hypothesis be sufficient to sustain a whole generation alive for seventy or eighty years, and then suddenly collapse. The Life-Table, if this hypothesis were true, would be represented by the *parallelogram* in which the curve of the Life-Table is inscribed (Plate XLII. fig. 1).

By the hypothesis of DEMOIVRE † the rate of mortality is such, that at the age of 20 one in 66 living at the beginning dies before the end of the year, leaving 65, 64, 63, 62, 61 to enter on each year of age until at the age of 86 all are dead.

Upon this hypothesis the relative numbers living up to the age 86 form an arithmetical progression: and the deaths in the equal times are equal out of the diminishing numbers living. The rate of mortality increases on this hypothesis as age advances in the same ratio as $n-\frac{1}{2}:1$; where n is the difference between the actual age x and 86. It is called the complement of life. The Life-Table, upon this hypothesis, has equal decrements, and might be represented on Plate XLII. fig. 1, by drawing a diagonal line through the parallelogram. Its deviation from the true curve on this scale is evident; but it is also evident that a series of straight lines, which would nearly represent the true curve, may be drawn from point to point of all the ordinates.

If the causes of death act with equal intensity at all ages, they may be represented by any simple external cause, destroying an equal *proportion* of the numbers living in equal intervals of time. Thus, if 1600 men were distributed equally over ground where

* By this 15 and *under* 25 years of age is understood, and so in all similar cases.

† See Treatise of Annuities on Lives, Preface to 2nd Edition.

they were exposed to certain dangers represented by successive discharges of musketry which at every discharge shot down one-half of the numbers remaining, they would be reduced successively from 1600 to 800, to 400, to 200, to 100, to 50, and so on *ad infinitum*, if a fraction of a living man could be conceived: the numbers living at each year of age in a Life-Table would not decrease at *these rates*, but they *would decrease* at a constant rate if the dangers at every stage of life remained *constant* and equally *great*. The numbers of the living at successive ages would be in geometrical progression, and would be represented by the ordinates of the logarithmic curve.

The law of mortality can only be derived from observation, and it is found to be less simple than either of these hypotheses implies. It can, however, be represented nearly by equations at different periods of age. Upon inspecting Table A (p. 864), it will be seen that at the age 55—65, which may be represented by the exact age 60, the mortality is such, that 2162 women die in a year out of a number equal to 100,000 living a year; and the mortality, which is the ratio of the dying to the living in a unit of time, here set down as a year, is therefore $m=\cdot02162$. Again, the mortality at the age of 70 is ·04992; at the age of 80 it is ·11866, and at the age of 90 it is ·26711. The mortality increases rapidly, and is more than doubled every ten years. The four numbers differ little from the terms of a geometrical progression, the logarithms of which have a constant difference. Let the rate at which the mortality increases be r, and $r^{10}=2\cdot3116$, and the first term (m) be ·02177; then a series of numbers will be formed differing little from those which express the value of m at decennial intervals of age.

Values of m at the precise age x.—*Females.*

Age (x).	60.	70.	80.	90.
By observation . . .	·02162	·04992	·11866	·26711
By hypothesis . . .	·02177	·05033	·11633	·26891

Note.—It may be assumed that m at 60 is the mean value of m in its range from $m_{59\frac{1}{2}}$ to $m_{60\frac{1}{2}}$; and so in other cases.

The *annual rate* of the increase of m from the age of 55 to 95 is $r=1\cdot0874$; and if m is the mortality at any age after 55, then $m_z=mr^z=$ the mortality at z years after the age at which m is taken. The common logarithm of r is $=\lambda r=\cdot03639$.

The mortality (m) of males at corresponding ages is higher than the mortality of females; but the rate of increase as age advances is nearly the same.

The value of m for females at the age of 20 is ·00765, and the mortality increases at the rate of nearly one-seventh part every ten years. The exact value of r is 1·0149, and $\lambda r=\cdot006423$.

Values of m.—*Females.*

Age.	20.	30.	40.	50.
By observation . . .	·00765	·00894	·00998	·01192
By hypothesis . . .	·00760	·00882	·01022	·01185

By these observations in the healthy districts the mortality (m) of men at the ages 15 to 45 is lower than the mortality of women at the same ages; yet during that period the rate of increase r is nearly the same for the two sexes. From the age of 40 to 50, and 50 to 60, the mortality of males increases at a rate intermediate between the rates of manhood and mature age.

Females.

Limits of ages.				
15 to 55	or	20 to 50	$r=1{\cdot}0149$	$\lambda r=\cdot 00642$
55 to 95	or	60 to 90	$r=1{\cdot}0874$	$\lambda r=\cdot 03639$

Males.

15 to 45	or	20 to 40	$r=1{\cdot}0148$	$\lambda r=\cdot 00640$
55 to 95	or	60 to 90	$r=1{\cdot}0874$	$\lambda r=\cdot 03640$

The subjoined Table exhibits the series of values for m derived from the hypothesis of two constant rates, and from direct observation. The values of r for females may be evidently applied to males in every period, except in the ten years of age, 40 to 50.

Mortality (m) of *males* and *females*, (1) derived from observation, and (2) from the *hypothesis* that m increases at the preceding rates.

Precise age.	Annual Mortality to 100 constantly living at each age (m).			
	Males.		Females.	
	By observation.	By hypothesis.	By observation.	By hypothesis.
20	·691	·696	·765	·760
30	·818	·807	·894	·882
40	·928	·935	·998	1·022
50	1·273	1·083	1·192	1·185
60	2·294	2·329	2·162	2·177
70	5·486	5·385	4·992	5·033
80	12·817	12·451	11·866	11·633
90	28·350	28·785	26·711	26·891
100	40·000 ?	66·550 ?	45·000 ?	62·160 ?

The observations on the numbers living and dying of the age of 95 and upwards are exceedingly uncertain; and it is probable that many of the persons believed to be 100, &c., are really persons five or ten years younger; so that these values of m_x, by the hypothetical method, are probably as correct as the direct numbers.

I shall now notice briefly the application of this hypothesis, first suggested by Mr. Gompertz, and applied by him to the interpolation of the Northampton and other Tables*. Mr. Edmonds, in 1832, extended the "Theory," and applied it to the construction of three Life-Tables†. He gave an elegant formula, similar in principle to that of Mr. Gompertz, from which the curve of a Life-Table can be deduced, upon the above hypothesis.

* Philosophical Transactions, 1825, paper by B. Gompertz, Esq., F.R.S.

† Life-Tables founded upon the discovery of a Numerical Law regulating the existence of every Human Being, &c. By T. R. Edmonds, B.A., 1832.

In the equation $\frac{s}{t}=v$, where *s* indicates space, *t* time, *v* velocity, the units of measure must be fixed before numbers can be inserted in the general expression; and then *v* will express, in the measure that has been applied to space, the number of such units of space described in *one* unit of time. Here *v* is a ratio; it is the rate at which the body moves: and in the same manner *m*, in the equation $\frac{d}{l}=m$, is the *rate of dying*, that is, as I shall express it, the *mortality*; or it is the ratio of the dying to the living in a given unit of time, the time during which the deaths occur being of precisely the same duration as the time during which the living are under observation,

l (living during 1 year) : d (dying during a year) :: 1 (year of life) : m.

If for *l* the number 100,000 is substituted, it is assumed that immediately a death occurs another life is substituted; and as the time is a year, then 760 will represent the value of *d* at the age 20, according to the preceding Table; $\therefore m=\cdot00760$. If the *time*, instead of *one year*, be the *thousandth part* of one year, then $m=\cdot0000076$; and if the time be infinitely short, *m* will be infinitely small: *m* is a ratio; the quantity of life existing during the time is represented by 1, and the quantity of life destroyed by a fraction, *m*. Whether the life inheres in the first organic molecule after conception, in the infant, or in the man, the vital action has a certain force of continuance, which is constantly varying; and the amount of this *force* that is *extinguished* at a given instant of time will be represented by the force of mortality, namely, by *m* at that instant. Then let the age $x=z+a$, where *a* represents the number of years up to the age at which a given rate (*r*) of increase of *m* begins; then $z=x-a$. And the mortality at any instant of age, in an instant of time at the end of *z* years or parts of years, will be mr^z. Now let *y* represent the living at that precise age; then the decrement of *y* in an infinitely short time will be $-dy=ymr^z dz$; the *dy* being negative as it is taken in a direction opposite to that in which the ordinate *y* of the curve is assumed to be drawn. Transferring *y* to the other side of the equation, this becomes $-\frac{dy}{y}=mr^z dz$; and integrating both sides, we have ($\lambda_, y$ being put for the hyperbolic logarithm of *y*, and $\lambda_, c$ for the difference between the constants of the two integrals)—

$$\lambda_, c-\lambda_, y=\lambda_, \frac{c}{y}=\frac{mr^z}{\lambda_, r}; \quad \ldots \quad (1.)$$

$$\therefore \quad \lambda_, y=\lambda_, c-\frac{mr^z}{\lambda_, r}, \quad \ldots \quad (2.)$$

and

$$\lambda_, c=\lambda_, y+\frac{mr^z}{\lambda_, r}. \quad \ldots \quad (3.)$$

When *z* is made zero, let $y=1$; then $\lambda_, y$ will also disappear, and $\lambda_, c=\frac{m}{\lambda_, r}$. Upon substituting this value of $\lambda_, c$ in equation (2.), it becomes

$$\lambda_, y=\frac{m}{\lambda_, r}-\frac{mr^z}{\lambda_, r}=\frac{m}{\lambda_, r}(1-r^z). \quad \ldots \quad (4.)$$

Upon passing to the numbers, equation (4.) becomes

$$y=\varepsilon^{\frac{m}{\lambda_{\prime}r}(1-r^{z})}=\text{the value of } y \text{ (taken as 1 at the origin) at the end of } z \text{ years.}$$

Let λ denote the common logarithm with the base 10; then $\lambda_{\prime}y=\frac{\lambda y}{k}$, where k is the modulus of the common system of logarithms; as also

$$\lambda_{\prime}c=\frac{km}{\lambda r}, \quad \text{and} \quad \frac{mr^{z}}{\lambda_{\prime}r}=\frac{kmr^{z}}{\lambda r}.$$

Equation (2.) becomes, after the required substitutions,

$$\frac{\lambda y}{k}=\frac{km}{\lambda r}-\frac{kmr^{z}}{\lambda r}$$

and

$$\lambda y=\frac{k^{2}m}{\lambda r}(1-r^{z}); \quad \ldots\ldots \quad (5.)$$

so the equation becomes finally

$$y=10^{\frac{k^{2}m}{\lambda r}(1-r^{z})}. \quad \ldots\ldots \quad (6.)$$

This is the form given by Mr. EDMONDS, and is convenient for use.

By making z successively 1, 2, 3, ... up to any number less than the number of years of age within which r remains constant, the number l_x being known, the number living at any other age within that range will be obtained by multiplying l_x by the corresponding value of y. Thus, if y_{10} is the value of y when $z=10$ in equation (6.); then putting l_{20} for the numbers living at the age 20, the living at the age 30 will be $y_{10}\times l_{20}=l_{30}$.

This hypothesis does not express the facts deduced from the observations exactly. If m_z could be expressed exactly over more than 20 years by $m_z=m_0r^{z}$, the first differences (δ^1) of the logarithms in the series following would in a certain number of cases be equal.

FEMALES in HEALTHY DISTRICTS of England.

Precise age.	Annual rate of mortality.	Logarithms of the annual mortality.	First decennial differences of λm_x.	Second decennial differences of λm_x.
x.	m*.	λm.	δ^1.	δ^2.
20	·00765	$\bar{3}$·8835	·0677	—·0197
30	·00894	$\bar{3}$·9512	·0480	·0290
40	·00998	$\bar{3}$·9992	·0770	·1817
50	·01192	$\bar{2}$·0762	·2587	·1047
60	·02162	$\bar{2}$·3349	·3634	·0126
70	·04992	$\bar{2}$·6983	·3760	—·0236
80	·11866	$\bar{1}$·0743	·3524	—·1259
90	·26711	$\bar{1}$·4267	·2265	
100	·45000	$\bar{1}$·6532		

* Here, at the age 20, m is the mean mortality that rules over the age $19\frac{1}{2}$ to $20\frac{1}{2}$ years of exact time.

The inequalities in the second differences vary in every separate class of observations; but there is generally a tendency in the first and in the second differences to increase, over a certain extent of the series. The error of the hypothesis is slight if the rate of increase (r), of which λ ·00677 is the logarithm in the case in hand, is only assumed to remain uniform for the ten years 20 to 30, or for the one year 20 to 21. Now let the number living at the age 20 be represented by l_{20}, and the number living at the age 21 by l_{21}; then put $\frac{l_{21}}{l_{20}}=p_{20}$. Here it is evident that if l_{20} and p_{20} be known, l_{21} is determined immediately by the equation $l_{21}=l_{20}\times p_{20}$. But p_{20} is the value of y in the equation $y_1=10^{\frac{k^2m}{\lambda r}(1-r^z)}$, when z is put $=1$. Taking the numbers from Table A., we have $m=\cdot 00765$ at the precise age $20=(19\frac{1}{2}+20\frac{1}{2})\frac{1}{2}$; and $\lambda m=\bar{3}\cdot 8835130$; $\lambda r=\cdot 0067728$; and $\therefore$ $r=1\cdot 015717$; k is put for the modulus of the common logarithms, $\therefore$ $\lambda k^2=\bar{1}\cdot 2755686$; $k(\lambda r)$ is the complement of the *logarithm* of (λr).

λk^2	$\bar{1}\cdot 2755686$
λm	$\bar{3}\cdot 8835130$
$k(\lambda r)$	$2\cdot 1692317$
$\lambda(1-r)$	$\bar{2}\cdot 1963697$
$-\cdot 0033472$	$\bar{3}\cdot 5246830$
$\bar{1}\cdot 9966528$	

As the factor $(1-r)$ is negative it makes the exponent of 10 negative, and upon taking the complement of this the logarithm of y is found to be $\bar{1}\cdot 9966528$. This is also the logarithm of $p_{20}=\cdot 99232$; and it enables us to pass, in the construction of a Life-Table, from the living at the age of 20 to the living at 21. If we obtain the several values p_x at every year of age, the whole of the Life-Table can be constructed.

It will be found that p_x is always a fraction, and it does not differ very much from $1-m_x$. But while m_x* shows the *deaths* in a year out of a *unit* of *life* (which may consist of any *number* of individual *lives* constantly kept up), p_x shows how much out of a *unit of the same life* at the beginning of a year, the dead not being replaced, *survives a year* after the age x; and $1-p_x$ is the amount of loss which occurs in the same year out of a unit of life at its commencement. Thus, as $p_{20}=\cdot 99232$, it follows that $1-p_{20}=\cdot 00768$. In the same year of age 20 to 21 the mortality is $m_{20}=\cdot 00771$, or ·00003 more than $(1-p_{20})$. If the unit of life is made 100,000 living at the age 20, then 99232 will survive, and 768 will die in the ensuing year of age. But if it is assumed that the deaths take place at equal intervals, it may also be assumed that the number of lives (100,000) being constantly sustained, the accessions of 768 new lives take place at equal intervals, consequently that they are under observation half a year on an average, giving the equivalent of $\frac{768}{2}=384$ years of lifetime at the age 20 to 21;

* m serves to indicate the mean mortality in the year following the exact age x.

now out of this number (384) at that age *three* die when the mortality is m_{20}. This accounts for the difference of ·00768 and ·00771; the former occurring in a year out of a unit of life of which the waste is not replaced.

From these considerations it may be inferred that if m_x is known, p_x may be deduced from it upon the hypothesis of equal decrements through the year by the formula $p_x=\frac{1-\frac{1}{2}m_x}{1+\frac{1}{2}m_x}=\frac{2-m_x}{2+m_x}$. Thus m_{20} being ·0077072, we have $\frac{\cdot 9961464}{1\cdot 0038536}=\cdot 99232$*, as before. The λp_{20} by the previous method is $\bar{1}\cdot 9966528$, and by this method it is the same. By either of the methods the value of p_x may be deduced for the subsequent ages, and $p_{20}, p_{30}, p_{40} \ldots\ldots p_{90}, p_{100}$ will be obtained. These values are here given, and it will be seen that the results by the two methods are nearly identical at all ages, except the two last, when the observations themselves become less exact.

Females.

Age (x).	$\lambda p_x=\lambda y_1=10^{\frac{\kappa 2m}{\lambda r}(1-r)}$.	$\lambda p_x=\lambda\left(\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}\right)$.
20	$\bar{1}$·9966528	$\bar{1}$·9966527
30	·9960967	·9960967
40	·9956263	·9956264
50	·9946669	·9946676
60	·9902049	·9902073
70	·9773538	·9773557
80	·9463182	·9462643
90	·8809176	·8801776

It will be observed that the fraction $p=\frac{1-\frac{1}{2}m}{1+\frac{1}{2}m}$ approximates to $1-m$ as m becomes less; for upon developing it into a series, $p=1-m+\frac{1}{2}m^2-\frac{1}{4}m^3+\frac{1}{8}m^4\ldots$ And taking m infinitely small, the terms after the two first may be neglected.

The values of $m_0, m_1 \ldots\ldots m_5$ may be obtained by the method already described. But it rarely happens that the population living at each year of age is accurately enumerated at the Census; and besides inaccuracies of statement, the numbers living at each of the early years of age fluctuate considerably, so that the numbers of children living of each year of age in 1851 do not represent the average numbers living of those ages in the five years 1849 to 1853, for instance.

The following method is less exceptionable. It may be assumed *for this purpose* (1) that the births registered in the year 1848 represent the births in that year; (2) that the births are equally distributed over the years in which they occur, and consequently

* m at the precise age 20 is nearly ·00765. The increase in this mortality from the age 20 to $20\frac{1}{2}$, the middle of the year of age 20 to 21 is obtained by adding $\frac{1}{2}\lambda r$, as above given, to $\lambda m_{19\frac{1}{2}}$, that is, to the log of $(m_{19\frac{1}{2}}+m_{20\frac{1}{2}})\frac{1}{2}$; ∴ $m_{20}=\cdot 0077072$

$$\begin{array}{rl} \lambda m_{19\frac{1}{2}} & \bar{3}\cdot 8835130 \\ \frac{1}{2}\lambda r & 0\cdot 0033864 \\ \hline \lambda m_{20} & \bar{3}\cdot 8868994 \end{array}$$

(3) that the *mean date* of all *the births* in the two years 1848, 1849 was immediately before January 1, 1849. The *half* of the births in those two years will consequently represent pretty accurately the number of births out of which the deaths of children *under one year* of age happened in the year 1849. And the deaths and survivors can be followed by this method year by year, as is evident in the annexed scheme:—

Age		
0	$\frac{1}{2}$ (births 1848, 1849)=mean annual births of which the mean date is January 1, [1849.	
	minus deaths under age 1 in 1849	
1		=surviving on January 1, 1850.
	minus deaths age (1 to 2) in 1850	
2		=surviving on January 1, 1851.
	minus deaths age (2 to 3) in 1851	
3		=surviving on January 1, 1852.
	minus deaths age (3 to 4) in 1852	
4		=surviving on January 1, 1853.
	minus deaths age (4 to 5) in 1853	
5		=surviving on January 1, 1854.

By commencing with the mean number of births in the years 1849, 1850, and deducting the deaths, a similar series may be obtained; and thus a succession of similar series may be deduced, the mean of which will supply the ordinary series l_0, l_1, l_2, l_3, l_4, l_5 of a Life-Table.

These series are liable to various disturbances. If all the births are not registered, the *rate* of mortality is overstated. If all the deaths are not registered, or if the children are carried off as emigrants, the decrements of life are understated. The annual number of births fluctuates, and now increases in England; they are in excess also in the early months of the year. Several of the disturbances are slight, and some of them are in opposite directions. The results can also be, and have been, checked by the results of the other method. The value of m_7 and m_{12} are deduced by dividing the annual deaths at the ages 5 to 10 and 10 to 15 by the mean population at those ages. The interpolation of the series λp_x from λp_3 to λp_{20} succeeds; taking λp_3, λp_7, λp_{12}, and λp_{20} as the fixed points of the series, and λp_{12} being adjusted to allow for the turn of the curve.

The Tables A, B, and C supply the data from which the Life-Table of Healthy English Districts was deduced. One or two arithmetical examples of the application of the method adopted in the earlier ages are also supplied.

III. INTERPOLATION.

We have therefore determined the values of λp_x at certain ages. The values of λp_x at the intervening ages may be determined by changing the value of r, and making z successively 1, 2......10 in the formula (p. 846). They may also be interpolated for every year of age by the method of finite differences; and upon the whole this method is preferable to any other. The logarithms of p_x are required; and to them it will be convenient to apply the interpolation directly. Any number of differences beyond four becomes cumbersome, and it will be therefore sufficient to give the general formula, which can be employed in deriving the first of either four or three orders of differences.

Investigation of Formulæ—Intervals equal.

Let any numbers of a series be so related that u_n, the nth from the first, u_0, is determined by the equation (1.)—

$$u_n = u_0 \pm \frac{n}{1}\delta^1 + \frac{n(n-1)}{1.2}\delta^2 \pm \frac{n(n-1)(n-2)}{1.2.3}\delta^3 + \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\delta^4. \quad . \quad . \quad (1.)$$

δ^1, δ^2, δ^3, and δ^4*, the first differences of the four orders, are unknown; they can all be determined from any five values of u_n. Now let n be successively $1x$, $2x$, $3x$, $4x$; then the coefficients of u_0, u_{1x}, u_{2x}, u_{3x}, u_{4x} can be found, to give the values of δ^1, δ^2, δ^3, and δ^4 in four equations. But when x is ten or more the coefficients become large, and the numerical calculation laborious. It is therefore well to obtain the numerical values of δ^4, δ^3, δ^2, δ^1 in succession. Thus if the series is ascending or descending, the following are convenient forms. The upper rows of signs are used in the *ascending*, the lower rows in the *descending* series:—

$$\delta^4 = \frac{+u_{4x} - 4u_{3x} + 6u_{2x} - 4u_x + u_0}{x^4}. \quad . \quad . \quad (2.)$$

$$\delta^3 = \frac{\pm u_{3x} \mp 3u_{2x} \pm 3u_x \mp u_0}{x^3} \mp \tfrac{3}{2}(x-1)\delta^4. \quad . \quad . \quad (3.)$$

$$\delta^2 = \frac{+u_{2x} - 2u_x + u_0}{x^2} \mp (x-1)\delta^3 - \frac{(7x^2-18x+11)}{12}\delta^4. \quad . \quad . \quad (4.)$$

$$\delta^1 = \frac{\pm u_x \mp u_0}{x} \mp \frac{x-1}{2}\delta^2 - \frac{(x^2-3x+2)}{6}\delta^3 \mp \frac{(x^3-6x^2+11x-6)}{24}\delta^4. \quad . \quad . \quad (5.)$$

It is necessary to be careful in deducing the successive values of δ from the values preceding; and before commencing their use their accuracy should be tested by inserting them in the checking equation,

$$u_{4x} = u_0 \pm \frac{4x}{1}\delta^1 + \frac{4x(4x-1)}{1.2}\delta^2 \pm \frac{4x(4x-1)(4x-2)}{1.2.3}\delta^3 + \frac{4x(4x-1)(4x-2)(4x-3)}{1.2.3.4}\delta^4. \quad . \quad . \quad (6.)$$

x may be any number. If only four terms are given, δ^3 is assumed to be constant; and δ^4 being 0, all the terms into which it enters disappear. The above formulæ, if this is borne in mind, are applicable when δ^4, δ^3, or δ^2 are assumed to be constant, and serve therefore to supply the differences when there are one, two, three, or four orders by the most expeditious method.

* It will be borne in mind that these imply first differences, or $\delta^1 u_0$, $\delta^2 u_0$, $\delta^3 u_0$, $\delta^4 u_0$.

In constructing the Life-Table, x was made 10 from the age of 20, and on inserting the numbers, the equations (2, 3, 4, 5, 6) became

$$\delta^4=\frac{\pm u_{40}\mp 4u_{30}\pm 6u_{20}\mp 4u_{10}\pm u_0}{10{,}000}. \qquad (7.)$$

$$\delta^3=\frac{\pm u_{30}\mp 3u_{20}\pm 3u_{10}\mp u_0}{1000}\mp 13\tfrac{1}{2}\delta^4. \qquad (8.)$$

$$\delta^2=\frac{\pm u_{20}\mp 2u_{10}\pm u_0}{100}\mp 9\delta^3 - 44\tfrac{1}{4}\delta^4. \qquad (9.)$$

$$\delta^1=\frac{\pm u_{10}\mp u_0}{10}\mp 4\tfrac{1}{2}\delta^2 - 12\delta^3\mp 21\delta^4. \qquad (10.)$$

The checking equation is

$$u_{40}=+u_0\pm 40\delta^1+780\delta^2\pm 9880\delta^3+91390\delta^4. \qquad (11.)$$

If three orders of differences are used, the checking equation is

$$u_{30}=+u_0\pm 30\delta^1+435\delta^2\pm 4060\delta^3. \qquad (12.)$$

After adding or subtracting any constant to or from a series of numbers, the differences remain the same; and if consecutive terms are multiplied or divided by the same factor, the differences are multiplied or divided by that factor. Thus $(b+a)-(c+a)=b-c$, and $ab-ac=a(b-c)$. Advantage is taken of these properties to reduce any one of the terms in the equations to *zero*.

Thus let the logarithms to be interpolated be the following—values of p_{20}, p_{30}, p_{40}, and p_{50}, taken from the column headed *males*, Table B; then they may, among other ways, be interpolated as follows:—

As $\bar{1}\cdot9969724$ is the contracted expression of $(\cdot9969724-1)$, we have

Age			
20	$\bar{1}\cdot9969724=-\cdot0030276$	(1) Multiplying each term by 10,000,000, that is, striking out the decimal point and the two adjoining ciphers, and (2) then subtracting from each 30,276, the values of $u_x=\lambda p_x$ to be operated on become	$u_0=-00000$
30	$\bar{1}\cdot9964260=-\cdot0035740$		$u_{10}=-\ 5464$
40	$\bar{1}\cdot9959051=-\cdot0040949$		$u_{20}=-10673$
50	$\bar{1}\cdot9943048=-\cdot0056952$		$u_{30}=-26676$

By inserting these values with their negative signs in the equations, and taking the upper signs, the three differences are found; that is,

$$\delta^3=-11\cdot049:\quad \delta^2=101\cdot991;\quad \text{and}\ \delta^1=-872\cdot7715.$$

The differences are now divided by 10,000,000, that is, ciphers are added to their left-hand side, so that the above decimal point may be moved seven places in that direction, and the operation may be thus commenced. By adding the differences successively to each other and to $\lambda p_{20}=\bar{1}\cdot 9969724$, the successive values are found of λp_{21}, λp_{22}, λp_{23}..... λp_{50} up to and including λp_{58} for males, where the series joins naturally the subsequent series, commencing at λp_{59}.

δ^3.	δ^2.	δ^1.	λp_x.
−·000,0011,0490	·000,0101,9910	−·000,0872,7715	$\bar{1}$·996,9724,0000
(constant)	·000,0090,9420	−·000,0770,7805	$\bar{1}$·996,8951,2285
		−·000,0679,8385	$\bar{1}$·996,8180,4480
			$\bar{1}$·996,7500,6095

In the actual operation the δ^3 is *subtracted* from δ^2, δ^2 from δ^1, and δ^1 from λp_x; it is therefore convenient to substitute for their present values the complements of δ^3 and δ^1, as thus all the series become additive.

As $\lambda l_{20}+\lambda p_{20}=\lambda l_{21}$, and $\lambda l_{21}+\lambda p_{21}=\lambda l_{22}$, and generally $\lambda l_x+\lambda p_x=\lambda l_{x+1}$, it is evident that the λp_x is the *first difference* of the series λl_x; and the whole series, λl_x, from λl_{20} to λl_{58}, may be formed as in the subjoined example, where δ^3 becomes δ^4, δ^2 becomes δ^3, and so on.

Healthy Districts.—Males.

δ^4 (constant)

9·999,9988,9510

Age.	δ^3.	δ^2.	$\delta^1=\lambda p_x$.	$u_x=\lambda l_x$.
20	0·000,0101,9910	9·999,9127,2285	9·996,9724,0000	4·584,1951,2769
21	0·000,0090,9420	9·999,9229,2195	9·996,8851,2285	4·581,1675,2769
22	0·000,0079,8930	9·999,9320,1615	9·996,8080,4480	4·578,0526,5054
23			9·996,7400,6095	4·574,8606,9534
24				4·571,6007,5629

Note.—The four last figures in the decimal portion of the series λp_x and in λl_x may in practice be omitted.

The corresponding values of λp_x in the column headed Females, Table B, are interpolated in the same way. And the λp_{60}, λp_{70}, λp_{80}, and λp_{90} are interpolated by the same methods, the series being continued backwards to λp_{57} and forwards to λp_{105}; the actual observations of age after the age of 90 furnishing results less reliable than those thus obtained, which bring a generation of 100,000 to their last end in 107 years. The successive values of λp_x in the period from the age of 3 to the age of 19 inclusive, are derived from λp_3, λp_7, λp_{12}, and λp_{20}, which represent u_0, u_4, u_9, and u_{17}. As the terms of the series are here at unequal distances, the first differences cannot be derived from the preceding formulæ. The δ can in this and similar cases be derived from the proper equations by substituting figures for letters. But three literal equations supply formulæ for finding the three first differences from any four terms of series of the kind which have been discussed: u_0, which has a troublesome coefficient, can always be reduced to *zero*, and is therefore omitted. The first given term being u_0, let the second u_x be the xth from u_0, and u_y be the yth, u_z the zth from u_0. Here $x<y<z$. Then the following equations give the differences*:—

$$\delta^3=\frac{6\left\{(y-x)\frac{u_z}{z}-(z-x)\frac{u_y}{y}+(z-y)\frac{u_x}{x}\right\}}{(y-x)\{(z-1)(z-2)-(y-1)(y-2)\}-(z-y)\{(y-1)(y-2)-(x-1)(x-2)\}} \quad (13.)$$

$$\delta^2=\frac{2}{y-x}\left\{\frac{u_y}{y}-\frac{u_x}{x}-\{(y-1)(y-2)-(x-1)(x-2)\}\frac{\delta^3}{6}\right\} \quad (14.)$$

$$\delta^1=\frac{u_x}{x}-(x-1)\frac{\delta^2}{2}-(x-1)(x-2)\frac{\delta^3}{6}. \quad (15.)$$

By making $y=2x$, and $z=3x$, these equations assume the same forms as equations (3.), (4.), (5.), with the term δ^4 struck out.

Putting $x=4$, $y=9$, and $z=17$, the three preceding equations become those which were actually used in constructing the series p_3 to p_{19}: u_0 is reduced to zero and is not used.

$$\delta^3=\frac{45u_{17}-221u_9+306u_4}{13260}. \quad (16.)$$

$$\delta^2=\frac{4u_9-9u_4-300\delta^3}{90}. \quad (17.)$$

$$\delta^1=\frac{u_4-6\delta^2-4\delta^3}{4}. \quad (18.)$$

Checking equation.

$$u_{17}=u_0+17\delta^1+136\delta^2+680\delta^3. \quad (19.)$$

* A useful Table in applying the above formulæ.

x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.	x.	$(x-1)(x-2)$.
20	342	30	812	40	1482	50	2352
21	380	31	870	41	1560	51	2450
22	420	32	930	42	1640	52	2550
23	462	33	992	43	1722	53	2652
24	506	34	1056	44	1806	54	2756
25	552	35	1122	45	1892	55	2862
26	600	36	1190	46	1980	56	2970
27	650	37	1260	47	2070	57	3080
28	702	38	1332	48	2162	58	3192
29	756	39	1406	49	2256	59	3306

Table of first differences in the Life-Table of Healthy Districts of England.

Age x.	λl_x.	$\lambda p_x = \delta^1$.	δ^2.	δ^3.	δ^4.
Males.					
3	4·631,5849,0000	9·993,2422,0000	0·001,2416,1260,934	9·999,8012,4393,666	0·000,0141,9648,567
20	4·584,1951,2769	9·996,9724,0000	9·999,9127,2285	0·000,0101,9910	9·999,9988,9510
59	4·403,7768,0454	9·990,6137,0980	9·998,9756,9020	9·999,9704,0800	9·999,9843,4520
60	4·394,3905,1434	9·989,5894,0000	9·998,9460,9820	9·999,9547,5320	9·999,9843,4520
Note.—The last series p_x was carried backwards from λp_{60} to λp_{59}.					
Females.					
3	4·623,2586,0000	9·993,2928,0000	0·001,2164,1598,794	9·999,7874,2556,561	0·000,0170,4566,365
20	4·570,6868,3846	9·996,6528,0000	9·999,9241,5455	0·000,0060,2930	9·999,9994,2530
57	4·405,2189,6826	9·992,9332,3725	9·999,0836,2675	0·000,0123,2100	9·999,9838,1950
60	4·381,2818,8126	9·990,2049,0000	9·999,0720,4825	9·999,9637,7950	9·999,9838,1950
Note.—The last series p_x was carried backwards from λp_{60} to λp_{57}.					

(20.)

A series of the form $v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2}$ is required in rendering the Life-Table applicable to the solution of questions in Annuities and Life Insurance.

The logarithms of the series are obtained by making the first term of the new series, $\lambda(v^x l_x)$, and the first term of the first order of differences $\lambda(vp_x) = \lambda v + \lambda p_x = \delta^1$, the δ^2, δ^3 and δ^4 of the original series remaining unchanged. Taking the interest of money at 3 per cent. $v = \frac{1}{1\cdot 03}$; and $\lambda v = \bar{1}\cdot 9871627,753$.

The derivation of the new series from this value of λv, and from the above Table (males), is shown in the annexed example. Any value of v^x may be introduced in the same way.

$$\delta^4 = 9\cdot 9999988,951$$

Age.	δ^3.	δ^2.	$\lambda(vp_x) = \delta^1$.	$u_0 = \lambda(l_x v^x)$.
20	0·0000101,991	9·9999127,2285	9·9841351,7530	4·3274506
	·0000090,942	·9999229,2195	·9840478,9815	·3115858
		·9999320,1615	·9839708,2010	·2956337
			·9839028,3625	·2796045
				·2635074

In describing the first English Life-Table, I ventured to express the belief that the chances of life may ultimately be calculated by Mr. BABBAGE's machine*. Mr. BABBAGE's conception has been realized in the original and ingeniously constructed machine of the Messrs. SCHEUTZ, which was favourably reported upon by a committee of the Royal Society. The first differences to be inserted in the machine can be immediately deduced from those given above; and we may hope ere long to see the logarithms of Life-Tables, for single and for joint lives, printed from types cast in moulds stamped by the machine now in the course of construction by the Messrs. DONKIN, for Her Majesty's Government, at the instance of the Registrar-General.

* Letter to the Registrar-General, in Appendix (p. 352) to his Fifth Annual Report, year 1843.

IV. CONSTRUCTION OF THE COLUMNS d_x, l_x, L_x, P_x, Q_x, Y_x, AND NOTICES OF SOME OF THEIR PRACTICAL APPLICATIONS.

The series l_x has been constructed; and from that series others are deduced to complete the Life-Table, consisting now of six columns.

(1.) $d_x=l_x-l_{x+1}=$number of deaths in the year of age following, out of l_x alive at the age x. By taking x successively at 0, 1, 2, 3, ... to the last age in the Table, the numbers *dying* in every year of age are obtained. The numbers dying of the age x and under the age l_{x+n} are immediately derived from the column l_x; as (2.) $l_x-l_{x+n}=d_x+d_{x+1}\dots d_{x+n-1}$. When $x+n>\omega=$ the oldest age in the Table, $l_x=d_x+d_{x+1}\dots+d_\omega$.

(3.) $L_x=l_x+l_{x+1}\dots\dots+l_\omega$. The series is formed by the successive addition of the series l_x, from l_ω upwards.

(3 *a*.) $L_x-L_{x+n}=L_{x|n}=l_x+l_{x+1}\dots+l_{x+n-1}$.

(4.) $\left.\begin{array}{l}P_x\ =l_{x+1}+\frac{1}{2}d_x\\ P_x\ =l_x\ \ -\frac{1}{2}d_x\end{array}\right\}$ and (5) $P_x=\frac{l_x+l_{x+1}}{2}$.

$P_{x+1}=l_{x+1}-\frac{1}{2}d_{x+1}=l_{x+2}+\frac{1}{2}d_{x+1}$.

The series in column P_x is constructed from the two columns l_x and d_x, or from the single column l_x, as $2P_x=l_x+l_{x+1}$; and $\therefore P_x=\frac{l_x+l_{x+1}}{2}$, $\therefore l_x=2P_x-l_{x+1}$; so, conversely, the series l_x can be constructed from the series P_x. The P_x is assumed to represent the population, as expressed by the Life-Table, living at the age x and under the age $x+1$. Thus $P_{20}=$ the population of the age 20 and under 21 years.

By substituting the successive values of P_x in the equation (5*a*), $P_x+P_{x+1}\dots P_{x+n}$, we have $\frac{1}{2}l_x+l_{x+1}\dots+l_{x+n}+\frac{1}{2}l_{x+n+1}$.

(6.) $Q_x=P_x+P_{x+1}+P_{x+2}\dots P_{x+n-1}+P_{x+n}\dots+P_\omega\dots\dots$

$Q_{x+n}=P_{x+n}+P_{x+n+1}+P_{x+n+2}\dots+P_\omega$.

(7.) $\therefore\ Q_x-Q_{x+n}=Q_{x|n}=P_x+P_{x+1}+P_{x+2}\dots P_{x+n-1}$. The column Q_x is constructed by adding up the column P_x, and transferring the successive sums to the column Q_x.

By substituting for the series P_x its values in l_x, we have

(8.) $Q_x=\frac{1}{2}l_x+l_{x+1}+l_{x+2}\dots\dots+l_\omega$.

And by again substituting for the series l_x its corresponding values in d_x, we have

(9.) $Q_x=\frac{1}{2}d_x+1\frac{1}{2}d_{x+1}+2\frac{1}{2}d_{x+2}\dots+(\omega+\frac{1}{2})d_\omega$.

(10.) Thus Q_x is equal to the numbers dying in each year of age after the age x, multiplied by the time (expressed in years and fractions of a year) that they have respectively lived over that age; and if $x=0$, then $Q_0=\frac{1}{2}d_0+1\frac{1}{2}d_1+2\frac{1}{2}d_2\dots(n+\frac{1}{2})d_{x+n}$, when $(x+n)$ becomes $>\omega$.

(11.) This column Q_x represents, therefore, two distinct orders of facts: it represents the sum of the number of years that will be lived after the age x by the l_x persons then living, and $\therefore\ \frac{Q_x}{l_x}=$ the mean after-lifetime; of which $\frac{Q_{x|n}}{l_x}$ will be enjoyed before the age $x+n$ is attained, and $\frac{Q_{x+n}}{l_x}$ after the age $x+n$ is attained. At birth the mean after-lifetime is $\frac{Q_0}{l_0}$, the unit here being one year of individual life.

(12.) Q_x also represents the sum of the numbers of men or women living at all ages over the age x, out of Q_0 living at all ages, as Q_x is in all cases the sum of the numbers living in each year of age, represented by the series P_x. The unit is here an individual man.

(13.) Thus, on referring to Plate XLII. fig. 1, the lifetime of 100,000 children born simultaneously may be represented by 100,000 parallel lines, drawn from AB horizontally in the direction of CD until they cut the curved line BC. And Q_0 is the sum of these lines expressed in the linear units of the scale on the line AC; so $\frac{Q_0}{l_0}=\frac{Q_0}{100{,}000}=\frac{4{,}899{,}665}{100{,}000}=48{\cdot}99665$; the mean length of those lines = the number of years of mean lifetime.

It will be observed that in this Table, instead of 100,000 lines, these lines are thrown into 106 groups, each comprising the variable number of lines terminating in each of 106 intervals numbered on the line AC, and representing years of age. And in these short intervals it is assumed that the mean length of the lines terminating in the eleventh interval (10 to 11) is represented by $10\frac{1}{2}$, and so on.

The relative numbers of persons living simultaneously at each interval of age will also be represented in the same Plate, fig. 1, by 106 successive vertical lines, raised from nearly the centre of each interval between the ordinates on the line AC, and measured in units of which the line AB contains 100,000. The same lines bound the figure representing the two orders of facts; and the numerical units expressing the aggregate length of the vertical lines equal in amount the units expressing the aggregate length of the horizontal lines expressed in the horizontal units.

(14.) I will now explain briefly the nature of the column Y_x, which I have added to the Life-Table*. The Life-Table (column P_x) exhibits a representative population, such as would be constituted by separating every year 100,000 births as they occurred,

* See paper in Appendix to Registrar-General's Sixth Annual Report, pp. 544–552.

Extract from the Registrar-General's Sixth Annual Report (1845), p. 528.

"*Note.*—HALLEY'S Table (1693) contained the column P. JOHN SMART made 1000 "born" the basis of his Table (1738), and introduced the columns d and l. SIMPSON adopted SMART'S form of Table, which was followed by KERSSEBOOM (1738), DEPARCIEUX (1746), PRICE (1773), and MILNE (1815). The columns $S.y$, y and Δy in DUVILLARD'S 'Loi de Mortalité (en France) dans l'état naturel †,' correspond with the columns L, l, d in the new Table. The $S.y$ added by DUVILLARD is our L and BARRETT'S column B; DUVILLARD'S short Table (p. 123) has the four columns d, l, P, Q for quinquennial or decennial ages, and the 'expectation of life.' MATHIEU'S Table II. is an expansion of the column Q of DUVILLARD'S short Table, and is that column for each year of age. In a recent report on the Bengal Military Fund, Mr. DAVIES has a Table (1) containing columns corresponding with the d, l, L, P, Q of the English Table, the 'Mortality per cent.,' and the 'Expectation of Life' at each age ‡."

I have in this paper employed d, l, L, instead of C, D, N, which have been formerly used by me and others, and should still be used where the factor v^x is introduced.

† Influence de la Petite Vérole, p. 161.

‡ See the note (A), p. 558.

and keeping them apart in a separate community, subject to a definite law of mortality. Any population living in the tabular proportions at each year of age may, for the sake of distinction, be called a normally constituted population.

The ages of the population represented by the Life-Table amount, in the aggregate, to Y_0 years; it is the aggregate *number of years which they have already lived*, and, singularly enough, it is also, if the law of mortality remain constant, *the number of years which they will live.* Thus Q_0 persons in such a population have lived on an average $\frac{Y_0}{Q_0}$ years; *that is their* MEAN AGE, and it is also their mean *after-lifetime.* Y_x is the number of years that Q_x persons have lived *over the age x*; and the mean age of such persons is $x+\frac{Y_x}{Q_x}$; their after-lifetime is $\frac{Y_x}{Q_x}$.

The series Y_x is formed by successively adding up a series of the form $\frac{1}{2}(Q_x+Q_{x+1})$, commencing at $x+1=\omega=$ the oldest age in the Table.

(15.) $\therefore\ Y_0=\frac{1}{2}Q_0+Q_1+Q_2\ldots+Q_\omega,$

$Y_x=\frac{1}{2}Q_x+Q_{x+1}+Q_{x+2}\ldots+Q_\omega.$

By substituting for Q_0, for Q_1, for Q_2, and so on, their values in P_x, it will be found that

(16.) $Y_0=\frac{1}{2}P_0+1\frac{1}{2}P_1+2\frac{1}{2}P_2+3\frac{1}{2}P_3\ldots.+(n+\frac{1}{2})P_n\ldots.+(\omega+\frac{1}{2})P_\omega.$

(17.) But the mean age of the persons (P_0) of the age of 0 and under 1 is nearly $\frac{1}{2}$; and so the series $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$ .... $(n+\frac{1}{2})$ expresses nearly the mean age of all the persons in the first (P_0), second (P_1), third (P_2), and $(n+1)$th (P_n) years of age, and so for all other ages; consequently the sum of the series (16) Y_0 is the sum of the ages of all the persons living contemporaneously, as they are represented in the Life-Table.

In like manner it is shown that

(18.) $$Y_x=\frac{1}{2}P_x+(1+\frac{1}{2})P_{x+1}+(2+\frac{1}{2})P_{x+2}\ldots\ldots+(\omega+\frac{1}{2}-x)P_\omega$$

is the sum of the number of years that the Q_x persons in the Table have lived over the age x. They have all lived x years; and consequently $x+\frac{Y_x}{Q_x}$ gives their average age precisely as $\frac{Y_0}{Q_0}$ gives the average age of the whole community.

(19.) It has been shown that Q_x expresses the number of years that l_x persons will live; in the same manner it may be shown that Q_{x+1} expresses the number of years that l_{x+1} persons will live; $\therefore$ (l_x+l_{x+1}) persons will live (Q_x+Q_{x+1}) years, $\therefore$ $\frac{1}{2}(l_x+l_{x+1})=P_x$ persons will live $\frac{1}{2}(Q_x+Q_{x+1})$ years. And the same may be demonstrated for each successive value of x.

But the sum of the series P_x is $Q_x=$ the number of persons living of all ages. And the sum of the series $\frac{1}{2}(Q_x+Q_{x+1})$ is $Y_x=$ the number of years that Q_x persons will live; $\therefore$ $\frac{Y_x}{Q_x}=$ the *mean after-lifetime* of all the persons living simultaneously of the age x and upwards. Thus by the Table D, 4,899,665 persons are living contemporaneously; their mean age is $\frac{Y_0}{Q_0}=\frac{166209701}{4899665}=33\cdot92$ years; and they will live on an average 33·92 years.

(20.) The Life-Table serves to determine the value of Life Annuities, the value of policies, and the premiums of insurance.

This is effected by introducing a new unit, such as £1, 1 franc, 1 dollar, or any other monetary unit. Thus if £1 is payable at each death, the series d_x will show the number of pounds falling due in each year of age; so if £1 is payable by each person on attaining the age x, and each subsequent year of age, the series l_x shows the number of *pounds* payable every year by the l_x persons; and N_x will be the number of pounds payable in the whole course of life after the age x: thus $\frac{N_x £1}{l_x}=$ the AVERAGE AMOUNT of an annuity of £1 payable on each life at and after the age x. The money-unit may be introduced into the other columns; and $\frac{Y_x}{Q_x}\cdot £1$ would show the AVERAGE AMOUNT payable under an annuity of £1 on each of Q_x lives. The *present value* of these future payments can always be determined by assuming a given rate of interest. The estimates thus obtained are also always read subject to the qualification that by hypothesis the *Life-Table* is based on a law of mortality actually to rule for a definite time in the population to which it is applied. The probability of the hypothesis is not here in question.

Under the same circumstances masses of mankind appear to experience, at the same ages, the same rates of mortality. Consequently if for several years d_x persons have died annually on an average out of l_x persons living at the beginning of the year, other things being equal, the probability that the same number will die out of l_x persons in a year to come is greater than any other that can be named, and the fraction expressing that probability is $\frac{d_x}{l_x}$. We know that d_x expressing the numbers dying in a year, l_{x+1} must express the numbers surviving as $l_{x+1}+d_x=l_x$. The chances may be represented by l_x balls; l_{x+1} *white* balls in an urn will represent the chances of living, d_x *black* balls in the same urn will represent the chances of dying. Now let each of l_x persons pay the sum z for a ticket, and each person that draws a *white ball* be entitled to £1. Before the drawing commences the value of each ticket is $\frac{l_{x+1}}{l_x}$; for l_x (the total chances) : l_{x+1} (the chances in favour of winning on one ticket) :: $1:\frac{l_{x+1}}{l_x}=z$.

Put $l_x=30{,}007$, and $l_{x+1}=29{,}647$; then $\frac{l_{x+1}\,.\,£1}{l_x}=\frac{29{,}647\,.\,£1}{30{,}007}=£\cdot 98802$. The amount of money to be paid on l_{x+1} white balls is £29,647, and $£\cdot 9802\times 30{,}007=z\,.\,l_x=£29{,}647$.

In like manner it may be shown that if £1 is paid to each person who draws a *black* ball, the value of each ticket is $\frac{d\,£1}{l_x}=y£1$; for $y\,.\,l_x\,.\,£1=d_x£1$, and £1 is to be paid on each of d_x tickets.

Should £1 be paid alike to those who draw white balls and to those who draw black balls, the value of a ticket will be equal to the sum of the two fractions expressing the several probabilities, namely,

$$\frac{l_{x+1}\,.\,£1}{l_x}+\frac{d_x£1}{l_x}=z+y=\frac{l_{x+1}+d_x}{l_x}£1=\frac{l_x}{l_x}£1=£1.$$

As one or other of the two kinds of balls *must by hypothesis be drawn*, and £1 is paid for each ball, the receipt of the £1 is certain: certainty is thus in all cases expressed by *unity*.

If every ball as it was drawn were replaced in the urn, although in 30,007 trials *white balls* were not actually drawn 29,647 times, black balls 360 times, still $\frac{29,647}{30,007}$ would express the probability of drawing a white ball, and the value of £1 contingent on that event, more accurately than any other fraction that could be named.

Again, if an urn contained by hypothesis an indefinite number of balls, out of which 29,647 white balls and 360 black balls were drawn and then replaced, the probability of again drawing a white ball on trial, and the value of £1 contingent on that event, would be expressed more accurately by $\frac{29,648}{30,009}$* than by any other fraction that could be named; past experience being by hypothesis the only means we have here of judging of the future.

Thus a Life-Table applicable to the case furnishes the fractions to determine the value of any sums of money dependent on the life or death of a given person, or a certain number of given persons in a given time.

The probability of living two years expressed by the fraction $\frac{l_{x+2}}{l_x}=\frac{l_x-(d_x+d_{x+1})}{l_x}$, is less than the probability of living one year.

Making n any number of years and fractional parts of years, the fraction $\frac{l_{x+n}}{l_x}$ will invariably express the probability of living n years after the age x. As n approaches zero the fraction will approximate to 1, the symbol of certainty; thus a person is more likely to live a day than a year, a minute than a day. As n increases l_{x+n} diminishes in value; and when $x+n$ expresses a year after the age ω in the Life-Table, $l_{\omega+1}$ is by hypothesis zero, $\therefore \frac{l_{\omega+1}}{l_x}=\frac{0}{l_x}=0$. The chance of living so long is expressed in this case by zero, the chance of dying in the time by 1, the symbol of certainty.

(21.) l_{x+n} expresses the number of chances in favour of surviving n years, and l_x-l_{x+n} the number of chances of dying in the same time, the sum of the two together (l_x) expressing the total number of chances. Thus the fraction $\left(\frac{l_{x+n}}{l_x}\right)$ expressing the probability of living a given time ranges from 1 to 0, and $\frac{l_x-l_{x+n}}{l_x}=1-\frac{l_{x+n}}{l_x}$, or the chance of dying in a given time also ranges from 1 to 0 as n varies. When the two fractions are equal $\frac{l_{x+n}}{l_x}=\frac{l_x-l_{x+n}}{l_x}$, then $l_{x+n}=l_x-l_{x+n}$, and $2l_{x+n}=l_x$, $\therefore l_{x+n}=\frac{l_x}{2}$.

To verify the equations, an age $x+n$ must be chosen at which l_{x+n} is exactly equal to $\frac{1}{2}l_x$. Thus by the Life-Table of healthy districts 100,000 children born alive are reduced to 50,851 in 58 years, and to 49,895 in 59 years; so the chances are rather in favour of

* The addition of 1 to the numerator, and of 2 to the denominator, may be neglected, when, as in this case, the numbers are large.

their living 58 years, as they are 50,851 to 49,149; upon the other hand, the chances of their living 59 years (49,895) are less than the chances 50,105 of their dying before attaining that age. Upon trial it will be found that the chances of living to and the chances of dying before $58\frac{851}{956}$ years$=58+\frac{50{,}851-50{,}000}{d_{58}}=58+\frac{851}{956}$ years, or about $58\frac{8}{9}$ years are nearly equal; hence this is called the *probable lifetime*, or *vie probable* by French writers, for $\frac{l_{58\frac{8}{9}}}{l_0}=\frac{1}{2}$. At the age 20 the probable lifetime is $47\frac{1588}{1638}$, nearly 48 years. The probable lifetime at every age is immediately seen by inspection.

(22.) V. THE THREEFOLD LIFE-TABLE—PERSONS, MALES, FEMALES.

The Life-Table is threefold. A Table having the six columns is made for males; another Table is separately made for females. The several columns of the two Tables incorporated together form the Table of persons which has 100,000, and may have any other number for its basis. The basis of the Male Table in the illustration is 51,125, while the basis of the Female Table is 48,875. In that proportion males and females were born in the districts. Under this arrangement the number of contemporaneous males and females living at each age in columns l_x is shown: thus 38,388 males and 37,212 females attain the age of 20; 17,145 males attain the age of 70, and 17,133 females attain the same age; at all ages under 71 the number of males exceeds the females; at the age of 71 and upwards the females exceed the males in number: and upon referring to the columns d_x, it will be seen that the males die off in greater numbers than females after the age of 42. The age after the second year at which the greatest number of deaths occurs is 75 in males, 76 in females.

These numbers all refer to the Life-Table for healthy districts.

Some of the other properties of the Life-Tables, admitting of innumerable applications in the solution of social phenomena, will appear in the following formulæ, which will be found useful in practice.

VI. USEFUL FORMULÆ.

The following formulæ will facilitate the use of the Life-Table. The figures must be taken from the Tables of Persons, of males or females, applicable to the case. The formulæ are general, and are applicable to any other Life-Table.

(23.) $\frac{d_x}{P_x}=m_x=$the rate of mortality in the year of age following the precise age x.

(24.) $\frac{d_x}{l_x}=\frac{l_x-l_{x+1}}{l_x}=1-\frac{l_{x+1}}{l_x}=$the probability that a person A of the age x, in average health, will die in the following year.

(25.) $\frac{l_{x+1}}{l_x}=p_x=\frac{l_x-d_x}{l_x}=1-\frac{d_x}{l_x}=$the probability that A, a person of the age x, will live a year; $\therefore 1-p_x=$the probability that A, age x, *will die in the year following*, as certainty of life $=1$.

(26.) $\frac{l_x - l_{x+n}}{l_x}$=the probability that A, age x, will die in the next n years.

(27.) $\frac{l_{x+n}}{l_x}$=the probability that A, of age x, will live n years.

(28.) Put $\frac{l_x}{2} = l_{x+n}$; and when l_{x+n} is taken at such an age as to fulfil the conditions of the equation, then n is the *probable lifetime*=*vie probable*=the time that it is an even chance a person of the age x will live.

(29.) $\frac{Q_x}{l_x} = A_x$=the mean *after lifetime*, or as it is often called, the *expectation of life*—an incorrect expression, which is rather applicable to the probable lifetime.

Note.—Upon DEMOIVRE'S hypothesis, the *probable lifetime*, that is the time that a person may fairly expect to live, his *expectation*, was the same as the mean after lifetime.

(30.) $G_x = x + A_x$=the mean age at death of persons who have already lived exactly x years.

(31.) $S = c\frac{Q_{x|n}}{l_x}$=the number of members of any Society between the ages x and $x+n$, which will be permanently sustained by c... annual admissions at the age x.

(32.) $c = \frac{Sl_x}{Q_{x|n}}$=annual recruits of the Society (S).

(33.) $\frac{Sl_{x+n}}{Q_{x|n}}$=annual members leaving the Society (S) on attaining the age $x+n$.

(34.) $\frac{Sl_{x|n}}{Q_{x|n}}$=annual deaths in such a Society (S).

(35.) $S\frac{Q_{x+n}}{Q_{x|n}}$=the aggregate number of persons living, who have left such a Society, as pensioners or otherwise.

In the following formulæ it is assumed that the population is normally constituted.

(36.) $\frac{Y_x}{Q_x} = A'_x$=the mean after lifetime of all persons of the *age* x *and upwards*.

(37.) $\frac{Y_x - Y_{x+n}}{Q_x - Q_{x+n}} = \frac{Y_{x|n}}{Q_{x|n}}$=the mean after lifetime of all persons of the age of x and under the age of $x+n$.

(38.) $c.\frac{Y_{x|n}}{Q_{x|n}}$=the number of persons of which a Society will *ultimately consist*, recruited by c annual additions of members in the tabular proportions between the age x and $x+n$.

(39.) $c\frac{Y_{x|n} - Y_{x+m|n}}{Q_{x|m}}$=the number of persons to which a Society joined by c persons of the tabular ages x and under $x+m$ would amount in n years. When $x+n>\omega$; this formula will be reduced to the same form as equation (38.). And when $x+m$, as well as $x+n>\omega$, the equation becomes the same as (36.).

VII. LIFE-TABLE OF THE SIXTY-THREE HEALTHIEST ENGLISH DISTRICTS.

Upon inquiry it was found that in many districts of England the mortality of the population did not exceed the rate of 17 annual deaths to 1000 living.

For the sake of convenience these were called healthy districts, consisting of sixty-four, or nearly a tenth part of the total registration districts of England and Wales, and inhabited by nearly a million of people: sixty-three of these districts have been taken as the basis of the new Life-Table, constructed according to the methods previously described.

It will be seen that these districts, generally conterminous with Poor Law Unions, are distributed over the various parts of the country. They comprise—*Hendon* (with Harrow*) (17), *Lewisham* (17), and *Bromley* (17) in the neighbourhood of London; *Hambledon* (16), *Dorking* (17), *Reigate* (16), and *Godstone* (17) on the southern slope of the Surrey hills; *East Ashford* (17) in East Kent, *Blean* (including Herne Bay) (17) between Canterbury and the sea; ten districts of Sussex—*Battle* (16) near Hastings, *Eastbourne* around Beachy Head (15), *Hailsham* (17), *Uckfield* (17), *East Grinstead* (17), *Cuckfield* (16), *Steyning* near *Brighton* (16), *Petworth* (17), *Worthing* (17), and *Midhurst* (17); seven districts of Hampshire—the *Isle of Wight* separated from the mainland by the sea (17), *Lymington* (17), *Christchurch* (16), *Ringwood* (17), *New Forest* (17), *Catherington* (17), and *Alresford* (17); *Wokingham* (17), and *Easthampstead* (16) in Berkshire, south of the Thames; *Ongar* (17) in Essex, east of Epping Forest; *Mutford* (17), including Lowestoft on the Suffolk coast; *Henstead* (17), south of Norwich; *Kingsbridge* (17), on the south coast of Devon; *Okehampton* (16); *Crediton* (17), *Barnstaple* (17), *Torrington* (17), *Bideford* (17), *Holsworthy* (16), stretching from the centre over Dartmouth and Exmoor, along the coast of the Bristol Channel; *Stratton* (17), *Camelford* (17), and *Launceston* (17), in the adjacent parts of Cornwall, and further south *St. Columb* (17); *Williton* (17) in Somerset, also on the Bristol Channel; *Winchcomb* (17), to the east of Cheltenham, and the Cotswold Hills around the sources of the Thames; *Kings Norton* (17) in Worcestershire, adjoining Birmingham; *Melton Mowbray* (17) in Leicestershire; *Southwell* (17) about Sherwood Forest, in the centre of Nottinghamshire; *Garstang* (16) in Lancashire, looking northward over Lancaster Bay; *Easingwold* (17) in the North Riding of Yorkshire, *Guisborough* (16) on the eastern coast north of Whitby; then follow five border districts of Northumberland on the southern face of the Cheviot Hills:—*Belford* (17), *Glendale* (15), *Rothbury* (15), *Bellingham* (17), *Haltwhistle* (16) (is omitted in the Table); *Longtown* (17) and *Brampton* (17) on the border, and *Bootle* (16) on the coast of Cumberland, the *East Ward* (17) of Westmoreland, *Haverfordwest* (17), on the western point of South Wales; *Builth* (16), *Corwen* (17), *Pwllheli* (17) on Carnarvon Bay, and *Anglesey* (17) complete the list. These districts, and others nearly equally healthy, have been thus described:—

"Such is the variety of the soil of England, that tested by the rates of mortality, the children reared out of a given number born, the longevity of the inhabitants, the free-

* The annual deaths to 1000 living of all ages inserted in parentheses are deduced from returns of the living at the censuses 1841 and 1851, and the deaths registered in the ten years 1841 to 1850. See Registrar-General's Sixteenth Report, pp. 141–153.

dom from common epidemics, or the immunity from cholera, Healthy Districts are found in nearly every county. Large tracts of country are, however, so much healthier than the rest, that they may be justly called Salubrious Fields; and it is remarkable that here the finest races of animals are bred. The north districts of Northumberland around the beautiful Cheviot Hills, covered with grasses, ferns, wild thyme,—extending from the region of the heaths to the rich cultivated land at their bases, touching each other, or intersected by narrow valleys; the districts extending from the Tees over the North and East Ridings of York to Leicestershire, Herefordshire, and parts of Shropshire; some of the districts of Gloucestershire about the Cotswold Hills; parts of Wales; North Devon, including Dartmoor and Exmoor; the Surrey and Sussex hills with the Southdowns,—have given names to the best breeds of sheep, fowls, cattle, and horses in the kingdom." * * * * * *

"The dry and most inland are not always the healthiest regions of the country. The salubrious fields are sometimes watered by running streams, and diversified by lakes; the dew is abundant; they are often veiled, not by infectious fogs, but by mists drawn from the sky as it breathes over them; the mountains rise above, the ocean rolls at the distance below them, as on the coast of Sussex, North Devon, the western region of Wales, extending under Snowdon and Cader Idris in a vast amphitheatre round Cardigan Bay; the lake land and moors of the North, rising between the Irish Sea and the German Ocean. The land is sometimes heathy, but may be covered by the sweetest herbage and bees feeding on the flowers: the cereal grains, the hop, the timber, are often of the finest quality; the animals are healthy, the native breeds are vigorous, and those fine varieties are produced at intervals, which men of the genius of BAKEWELL, ELLMAN, TOMKINS, COLLING, and O'KELLY make the permanent stock of the country. Industry and the army receive their best recruits from the population; while they get their worst from the people of the low parts of sickly towns. Agriculture has reclaimed many unhealthy districts on the plains, so that a considerable extent of the cultivated land is now in a state of comparative salubrity; and vast systems of drainage have subdued the noxious fens, although carried out less efficiently than is desirable, and interfered with by milldams on the rivers, descending like the Nene from the inland high lands*."

The sanitary condition of the people in these districts is, however, still in many respects defective.

CONCLUSION.

HALLEY first pointed out the financial applications of the Life-Table, and first calculated the values of life annuities. That branch of science, in the various forms of life insurance, has since received great developments. The new Table shows that the duration of life, among large classes of the population, by no means in unexceptionable sanitary conditions, exceeds the term of the ordinary Tables, and proves that life annuities cannot be sold advantageously by offices, or by the Government, to large classes of lives for less than the values deducible from the new Table.

A new branch of science has been developed since HALLEY'S day,—it is the science of Public Health. And here a new application of the Life-Table is found.

* Report to the Registrar-General on Cholera, pp. xcv, xcvi.

It is probable, upon physiological grounds, that man goes through all the phases of his natural development in a hundred years; and that the period of active life seldom extends beyond eighty years. But this is a very indefinite measure, as the rates of mortality, in all the intermediate ages, are left undetermined after it has been ascertained in what proportions men attain the extreme limits.

Generations of men, under all circumstances, die at all ages; but the proportions vary indefinitely under different conditions from a slight tribute to death each year, down to the point of extermination by pestilence. If we ascertain at what rate a generation of men dies away under the least unfavourable existing circumstances, we obtain a standard by which the loss of life, under other circumstances, is measured; and this I have endeavoured to determine in the Life-Table of English Healthy Districts. And recollecting that the science of public health was almost inaugurated in England by a former President of this Society*, who encouraged and crowned the sanitary discoveries of Captain COOK, I feel assured that it will receive with favour this imperfect attempt to supply sanitary inquirers with a scientific instrument.

In a subsequent paper I hope to be able to lay before the Society the mortality by different kinds of diseases at each age, as they have been deduced from the same series of observations.

HEALTHY DISTRICTS.

TABLE A.—Population, 1851. Deaths in the five years 1849 to 1853. Average Annual Mortality per cent., and Logarithms of the Mortality.

Ages.	Population.			Deaths.			Average annual mortality to 100 living (m).			Logarithms of the mortality (λm).		
	Persons.	Males.	Females.	Persons.	Males.	Females.	Persons.	Males.	Females.	Persons.	Males.	Females.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
All ages	996773	493525	503248	87345	43736	43609	1·753	1·772	1·733	$\bar{2}$·2436718	$\bar{2}$·2485599	$\bar{2}$·2388240
Under 5	130635	65700	64935	26361	14282	12079	4·036	4·348	3·720	$\bar{2}$·6059323	$\bar{2}$·6382536	$\bar{2}$·5705821
5—	122406	61733	60673	4209	2080	2129	·688	·674	·702	$\bar{3}$·8374062	$\bar{3}$·8285759	$\bar{3}$·8462102
10—	110412	56651	53761	2377	1087	1290	·431	·384	·480	$\bar{3}$·6340429	$\bar{3}$·5840519	$\bar{3}$·6811523
15—	181339	90066	91273	6603	3113	3490	·728	·691	·765	$\bar{3}$·8622801	$\bar{3}$·8396482	$\bar{3}$·8835130
25—	136892	65422	71470	5869	2675	3194	·857	·818	·894	$\bar{3}$·9332160	$\bar{3}$·9126300	$\bar{3}$·9512411
35—	108056	52734	55322	5208	2447	2761	·964	·928	·998	$\bar{3}$·9840521	$\bar{3}$·9675733	$\bar{3}$·9991985
45—	85244	42383	42861	5252	2698	2554	1·232	1·273	1·192	$\bar{2}$·0906909	$\bar{2}$·1048802	$\bar{2}$·0761886
55—	62857	31105	31752	7001	3568	3433	2·228	2·294	2·162	$\bar{2}$·3478365	$\bar{2}$·3606246	$\bar{2}$·3349327
65—	39453	18860	20593	10313	5173	5140	5·228	5·486	4·992	$\bar{2}$·7183350	$\bar{2}$·7392308	$\bar{2}$·6982734
75—	16737	7718	9019	10297	4946	5351	12·304	12·817	11·866	$\bar{1}$·0900631	$\bar{1}$·1077793	$\bar{1}$·0743066
85—	2614	1097	1517	3581	1555	2026	27·399	28·350	26·711	$\bar{1}$·4377287	$\bar{1}$·4525536	$\bar{1}$·4266838
95 and upwards	128	56	72	274	112	162	42·813	40·000	45·000	$\bar{1}$·6315706	$\bar{1}$·6020600	$\bar{1}$·6532125

Note.—The ages at death of 146 persons, viz. 123 males and 23 females, were not stated; in calculating the mortality they have been distributed proportionally over the several ages in the Table. The Table may be read thus: 136,892 persons, of whom 65,422 were males, 71,470 were females at the age of 25 and under 35, were enumerated in 1851; at the same ages, 5869, 2675 males and 3194 females, died in the five years 1849 to 1853; consequently the annual rates of mortality per cent. were ·857, ·818, and ·894.

* Sir JOHN PRINGLE.

Number of Deaths at five periods of Age in the Healthy Districts, in 1848 to 1855.

Years.	Ages.														
	Persons.					Males.					Females.				
	0.	1.	2.	3.	4.	0.	1.	2.	3.	4.	0.	1.	2.	3.	4.
1848.	2935	832	458	371	312	1678	442	244	204	162	1257	390	214	167	150
1849.	2932	858	541	427	292	1637	452	263	207	154	1295	406	278	220	138
1850.	2969	859	466	331	301	1676	453	231	164	144	1293	406	235	167	157
1851.	3185	932	543	341	288	1769	502	274	179	148	1416	430	269	162	140
1852.	3405	860	567	389	297	1913	446	273	206	140	1492	414	294	183	157
1853.	3370	946	554	376	287	1888	514	293	179	137	1482	432	261	197	150
1854.	3404	1047	601	386	311	1903	539	317	197	165	1501	508	284	189	146
1855.	3350	907	533	445	297	1948	483	257	230	156	1402	424	276	215	141

Number of Births in Sixty-three Healthy Districts of England, 1848 to 1855.

Years.	Persons.	Males.	Females.
1848	28679	14756	13923
1849	29128	14751	14377
1850	29699	15176	14523
1851	30163	15465	14698
1852	30370	15557	14813
1853	29214	15010	14204

Age.	Males.
	2)29,507=births in 1848 and 1849
0	14,754=births on January 1, 1849
1	13,117=living on January 1, 1850
2	12,664=living on January 1, 1851
3	12,390=living on January 1, 1852
4	12,184=living on January 1, 1853
5	12,047=living on January 1, 1854

Age.	Males.
0	1637=deaths in 1849
1	453=deaths in 1850
2	274=deaths in 1851
3	206=deaths in 1852
4	137=deaths in 1853

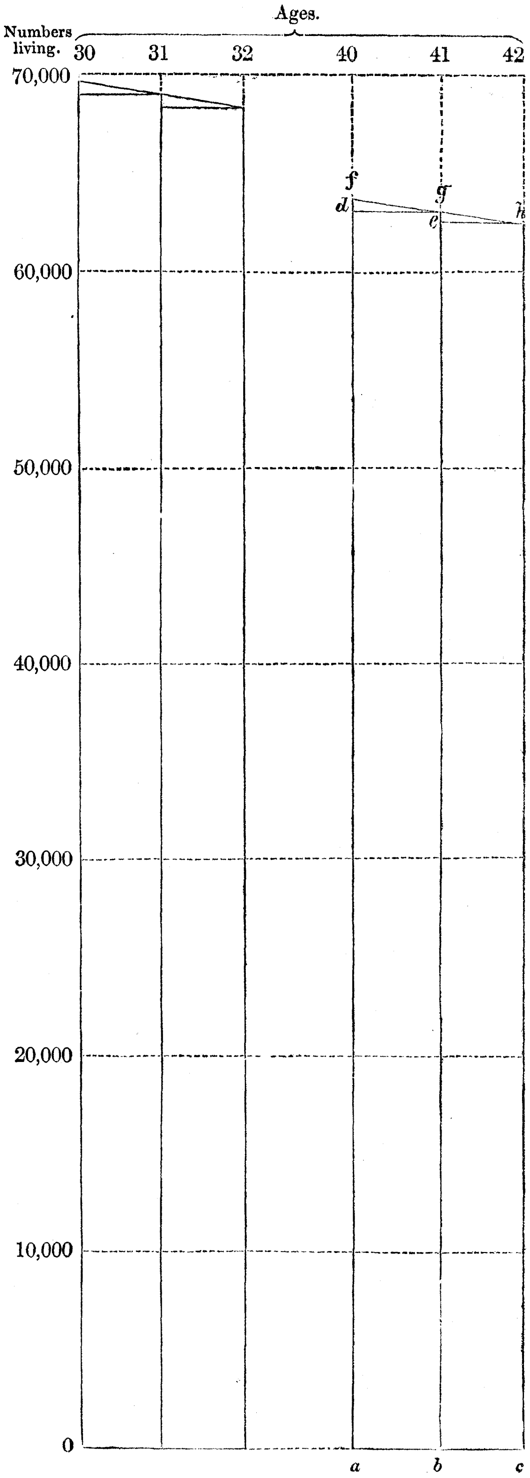
TABLE B.—The several values of λp_x on which the Life-Table of Healthy Districts is based: also the corresponding values of p_x and $(1-p_x)$.

Age x.	λp_x =logarithms of the probability of living one year after the age x.		p_x =probability of *living* a year.		$(1-p_x)$ =probability of *dying* in a year.	
	Males.	Females.	Males.	Females.	Males.	Females.
0	$\bar{1}\cdot9480215$	$\bar{1}\cdot9577796$	·88720	·90736	·11280	·09264
1	$\bar{1}\cdot9844929$	$\bar{1}\cdot9859276$	·96492	·96812	·03508	·03188
2	$\bar{1}\cdot9904341$	$\bar{1}\cdot9904679$	·97821	·97829	·02179	·02171
3	$\bar{1}\cdot9932422$	$\bar{1}\cdot9932928$	·98456	·98467	·01544	·01533
7	$\bar{1}\cdot9970729$	$\bar{1}\cdot9969512$	·99328	·99300	·00672	·00700
12	$\bar{1}\cdot9984539$	$\bar{1}\cdot9980197$	·99645	·99545	·00355	·00455
20	$\bar{1}\cdot9969724$	$\bar{1}\cdot9966528$	·99305	·99232	·00695	·00768
30	$\bar{1}\cdot9964260$	$\bar{1}\cdot9960967$	·99180	·99105	·00820	·00895
40	$\bar{1}\cdot9959051$	$\bar{1}\cdot9956263$	·99062	·98998	·00938	·01002
50	$\bar{1}\cdot9943048$	$\bar{1}\cdot9946669$	·98697	·98780	·01303	·01220
60	$\bar{1}\cdot9895894$	$\bar{1}\cdot9902049$	·97631	·97770	·02369	·02230
70	$\bar{1}\cdot9751357$	$\bar{1}\cdot9773538$	·94436	·94919	·05564	·05081
80	$\bar{1}\cdot9420680$	$\bar{1}\cdot9463182$	·87512	·88373	·12488	·11627
90	$\bar{1}\cdot8747315$	$\bar{1}\cdot8809176$	·74943	·76018	·25057	·23982

Note.—Age x is in this Table the precise age. Age 12 is applied frequently to all persons of the age of 12 and under the age of 13; but in this Table it applies only to persons of the precise age of 12 years, neither more nor less. The λp_7 was in both cases derived from the formula $\left(\frac{2-m}{2+m}\right)$. The λp_{12}, deduced from this formula, is for males $\bar{1}\cdot9983497$, and for females $\bar{1}\cdot9979153$; which may be regarded either as the constant or the mean values of λp_{10}, λp_{11}, λp_{12}, λp_{13}, and λp_{14}; but as these are the terminations of an ascending and a descending series, it is probable, and quite in conformity with other observations, that one, two, or more of these values will exceed the mean value. The logarithms of p_{12} adopted are given above; and the two arithmetical means of the five logarithms, λp_{10}, λp_{11}, λp_{12}, λp_{13}, and λp_{14}, resulting from the interpolation, are $\bar{1}\cdot9983688$ for males, and $\bar{1}\cdot9979435$ for females.

The values of λp_{20}, λp_{30} .... are derived from the formula $y_x=10^{\frac{k^2m}{\lambda r}(1-r^x)}$.

NOTE ON THE TWO HYPOTHESES.



Let b be the decrement of the ordinate y in a unit of time, then the decrement Δy of the ordinate in the time x, represented by the abscissa, will be $\Delta y = -bx$, on DEMOIVRE's hypothesis; and as it is always proportional to the time, it will be in an infinitely short time $dy = -bdx$.

Passing to the integral $y = c - bx$. And if $y = a$ at the origin when $x = 0$, $c = a$, $\therefore y = a - bx$. And if $b = 1$, then $y = a - x$. This evidently represents very closely short portions of the Life-Table curve; and the smaller x is taken, the nearer is the approximation to the corresponding value of y.

Again, let Δy be the decrement of the ordinate y in the indefinite time Δx represented by the abscissa; and let the mortality (m) represented by the ratio of the area $abfg$ to the area dfg be $\frac{d_0}{P_0} = m_0$. Let also m_0 increase at the rate r in a unit of time, so that $\frac{geh}{bcgh} = \frac{d_1}{P_1} = m_1 = m_0 r$, and generally within given limits $m_0 r^x = m_x$; then $\Delta y = -y m_x \Delta x$ nearly, Δx being any small portion of time.

The error increases as the time Δx is extended, from the circumstance that on the one hand m_x varies by hypothesis momentarily, and that y, from which the varying proportional part is taken, constantly grows shorter. But by passing to the limit and making the time dx infinitely short, m_x and y during that infinitely short time may be considered constant, and $dy = -y m_x dx$ will be the true decrement. Substituting $m_0 r^x$ for m_x, the equation becomes $dy = -y m_0 r^x dx$, from which the value of y can be derived, as before shown. For $\frac{dy}{y} = -m_0 r^x dx$, and integrating both sides $\lambda_\epsilon y = \lambda_\epsilon c - \frac{m_0 r^x}{\lambda_\epsilon r}$. Here λ_ϵ stands for the logarithm having ϵ for its base.

At the origin of the curve, when $x = 0$, let $y = 1$, and then $\lambda_\epsilon c = \frac{m_0}{\lambda_\epsilon r}$. Now substituting this value for $\lambda_{\iota}c$, we have $\lambda_{\iota}y=\frac{m_0}{\lambda_{\iota}r}-\frac{m_0 r^x}{\lambda_{\iota}r}$, $\therefore$ $\lambda_{\iota}y=\frac{m_0}{\lambda_{\iota}r}(1-r^x)$; and passing to the number, $y=\varepsilon^{\frac{m_0}{\lambda r}(1-r^x)}$. Putting k for the modulus of the common logarithm (λ) having 10 for its base, we have $\lambda_{\iota}y=\frac{\lambda y}{k}$, and $\lambda_{\iota}r=\frac{\lambda r}{k}$, $\therefore$ $\frac{\lambda y}{k}=\frac{km}{\lambda r}(1-r^x)$; or passing to the number, $y=10^{\frac{k^2m}{\lambda r}(1-r^x)}$.

Upon the one hypothesis, out of a generation of men an *equal quantity of life** is destroyed in equal times, out of diminishing quantities in existence, the *proportion* that perishes of the residual life constantly *increasing*.

Upon the other hypothesis, a *decreasing proportion* of the residual life is destroyed from birth down to the age of puberty; in the after ages, a *proportion increasing* at different rates is destroyed in equal times. The *quantity* of life *destroyed* in equal times may be the same, or different upon this hypothesis. And in very short intervals of age the differences between *the quantities* of *life destroyed* may be so inconsiderable, that they may be neglected.

The two hypotheses may be illustrated. Assume that at every beat of the heart an equal quantity of vital force on an average is consumed in excess of that produced; or if this does not happen at distant ages, assume that it happens during two consecutive years, two consecutive days, two consecutive pulses of a generation of men, and is represented by the deaths in the two intervals; this will give an idea of the first hypothesis.

The second hypothesis will be represented by assuming that, in addition to the existing force, a certain amount of vital force is produced, while a certain amount is also destroyed at every beat of the heart; the quantity destroyed exceeding the quantity produced in a diminishing ratio, and then in an increasing ratio; the proportional part destroyed being for this purpose always represented by the proportional number of hearts beating to the number of hearts ceasing to beat at every instant of age, among a generation of men. The respirations, the sensations, the secretions, nutrition, and all the vital acts may be conceived like the heart to influence the continuance of the vital force; implying here simply the force which sustains life.

* The quality or the intensity of life at different ages is purposely left out of consideration.

June 15, 1859.

Table B1.—LIFE-TABLE OF HEALTHY ENGLISH DISTRICTS.

Logarithms of the Numbers of Males and Females living at each year of age.

λl_x.				λl_x.			
Age. x.	Males.	Age. x.	Females.	Age. x.	Males.	Age. x.	Females.
0	4·7086364	0	4·6890835	55	4·4351998	55	4·4177773
1	4·6566579	1	4·6468631	56	4·4279544	56	4·4116015
2	4·6411508	2	4·6327907	57	4·4203212	57	4·4052190
3	4·6315849	3	4·6232586	58	4·4122719	58	4·3981522
4	4·6248271	4	4·6165514	59	4·4037768	59	4·3901691
5	4·6193109	5	4·6110606	60	4·3943905	60	4·3812819
6	4·6148376	6	4·6065737	61	4·3839799	61	4·3714868
7	4·6112225	7	4·6028950	62	4·3725154	62	4·3607637
8	4·6082954	8	4·5998462	63	4·3599518	63	4·3490765
9	4·6059001	9	4·5972658	64	4·3462281	64	4·3363727
10	4·6038946	10	4·5950094	65	4·3312678	65	4·3225837
11	4·6021511	11	4·5929497	66	4·3149786	66	4·3076249
12	4·6005560	12	4·5909763	67	4·2972528	67	4·2913951
13	4·5990100	13	4·5889960	68	4·2779668	68	4·2737774
14	4·5974279	14	4·5869326	69	4·2569814	69	4·2546384
15	4·5957387	15	4·5847269	70	4·2341418	70	4·2338287
16	4·5938855	16	4·5823368	71	4·2092775	71	4·2111825
17	4·5918259	17	4·5797373	72	4·1822024	72	4·1865180
18	4·5895314	18	4·5769202	73	4·1527146	73	4·1596372
19	4·5869878	19	4·5738947	74	4·1205968	74	4·1303259
20	4·5841951	20	4·5706868	75	4·0856157	75	4·0983537
21	4·5811675	21	4·5673396	76	4·0475228	76	4·0634741
22	4·5780527	22	4·5639166	77	4·0060534	77	4·0254242
23	4·5748607	23	4·5604237	78	3·9609277	78	3·9839252
24	4·5716008	24	4·5568665	79	3·9118498	79	3·9386819
25	4·5682808	25	4·5532498	80	3·8585083	80	3·8893831
26	4·5649078	26	4·5495779	81	3·8005763	81	3·8357013
27	4·5614874	27	4·5458546	82	3·7377111	82	3·7772929
28	4·5580244	28	4·5420830	83	3·6695542	83	3·7137979
29	4·5545223	29	4·5382656	84	3·5957318	84	3·6448405
30	4·5509835	30	4·5344046	85	3·5158541	85	3·5700284
31	4·5474095	31	4·5305013	86	3·4295159	86	3·4889532
32	4·5438005	32	4·5265566	87	3·3362962	87	3·4011904
33	4·5401557	33	4·5225708	88	3·2357583	88	3·3062992
34	4·5364730	34	4·5185435	89	3·1274500	89	3·2038228
35	4·5327494	35	4·5144739	90	3·0109034	90	3·0932880
36	4·5289808	36	4·5103606	91	2·8856349	91	2·9742056
37	4·5251620	37	4·5062016	92	2·7511453	92	2·8460701
38	4·5212864	38	4·5019942	93	2·6069196	93	2·7083599
39	4·5173467	39	4·4977353	94	2·4524273	94	2·5605372
40	4·5133342	40	4·4934212	95	2·2871223	95	2·4020479
41	4·5092393	41	4·4890475	96	2·1104426	96	2·2323219
42	4·5050512	42	4·4846093	97	1·9218108	97	2·0507729
43	4·5007579	43	4·4801012	98	1·7206337	98	1·8567982
44	4·4963465	44	4·4755172	99	1·5063024	99	1·6497793
45	4·4918029	45	4·4708506	100	1·2781926	100	1·4290811
46	4·4871119	46	4·4660943	101	1·0356640	101	1·1940526
47	4·4822570	47	4·4612404	102	0·7780608	102	0·9440265
48	4·4772210	48	4·4562807	103	0·5047118	103	0·6783194
49	4·4719852	49	4·4512061	104	0·2149296	104	0·3962318
50	4·4665301	50	4·4460074	105	9·9080117	105	0·0970476
51	4·4608349	51	4·4406743	106	9·5832396	106	9·7800351
52	4·4548778	52	4·4351962	107	9·2398792	107	9·4444460
53	4·4486358	53	4·4295620	108	8·8771808	108	9·0895160
54	4·4420848	54	4·4237598	109	8·4943792	109	8·7144646

The above Tables were calculated and stereoglyphed by Scheutz's Calculating Machine at the General Register Office, Somerset House. The impression was made by the machine on *papier maché* in the dry state. Sheet lead received the impressions in the original invention. The use of *papier maché* was suggested by Mr. W. Mattress, Overseer in the Firm of Messrs. Taylor and Francis. In the wet state, as it is used by stereotype founders, *papier maché* did not however succeed; but after several trials, it was found that dry *papier maché*, black-leaded, supplies a good mould for the stereotype metal.

Table C.—HEALTHY DISTRICTS.

Age.	Living at each age (l_x).			Dying in each year of age (d_x).			Age
x.	Persons.	Males.	Females.	Persons.	Males.	Females.	x.
0	100000	51125	48875	10295	5767	4528	0
1	89705	45358	44347	3005	1591	1414	1
2	86700	43767	42933	1885	953	932	2
3	84815	42814	42001	1305	661	644	3
4	83510	42153	41357	1051	532	519	4
5	82459	41621	40838	847	427	420	5
6	81612	41194	40418	682	341	341	6
7	80930	40853	40077	555	275	280	7
8	80375	40578	39797	459	223	236	8
9	79916	40355	39561	391	186	205	9
10	79525	40169	39356	347	161	186	10
11	79178	40008	39170	324	146	178	11
12	78854	39862	38992	319	142	177	12
13	78535	39720	38815	328	144	184	13
14	78207	39576	38631	350	154	196	14
15	77857	39422	38435	379	168	211	15
16	77478	39254	38224	414	186	228	16
17	77064	39068	37996	451	205	246	17
18	76613	38863	37750	489	227	262	18
19	76124	38636	37488	524	248	276	19
20	75600	38388	37212	552	267	285	20
21	75048	38121	36927	562	272	290	21
22	74486	37849	36637	571	277	294	22
23	73915	37572	36343	577	281	296	23
24	73338	37291	36047	583	284	299	24
25	72755	37007	35748	588	287	301	25
26	72167	36720	35447	591	288	303	26
27	71576	36432	35144	593	289	304	27
28	70983	36143	34840	595	290	305	28
29	70388	35853	34535	596	291	305	29
30	69792	35562	34230	598	292	306	30
31	69194	35270	33924	599	292	307	31
32	68595	34978	33617	599	292	307	32
33	67996	34686	33310	601	293	308	33
34	67395	34393	33002	601	293	308	34
35	66794	34100	32694	603	295	308	35
36	66191	33805	32386	604	296	308	36
37	65587	33509	32078	608	298	310	37
38	64979	33211	31768	610	300	310	38
39	64369	32911	31458	613	302	311	39
40	63756	32609	31147	618	306	312	40
41	63138	32303	30835	623	310	313	41
42	62515	31993	30522	630	315	315	42
43	61885	31678	30207	638	320	318	43
44	61247	31358	29889	645	326	319	44
45	60602	31032	29570	656	334	322	45
46	59946	30698	29248	666	341	325	46
47	59280	30357	28923	679	350	329	47
48	58601	30007	28594	692	360	332	48
49	57909	29647	28262	706	370	336	49

50	57203	29277	27926	722	381	341	50
51	56481	28896	27585	740	394	346	51
52	55741	28502	27239	758	407	351	52
53	54983	28095	26888	777	420	357	53
54	54206	27675	26531	798	435	363	54
55	53408	27240	26168	820	451	369	55
56	52588	26789	25799	843	467	376	56
57	51745	26322	25423	894	483	411	57
58	50851	25839	25012	956	501	455	58
59	49895	25338	24557	1040	542	498	59
60	48855	24796	24059	1123	587	536	60
61	47732	24209	23523	1205	631	574	61
62	46527	23578	22949	1281	672	609	62
63	45246	22906	22340	1356	712	644	63
64	43890	22194	21696	1430	752	678	64
65	42460	21442	21018	1501	789	712	65
66	40959	20653	20306	1571	826	745	66
67	39388	19827	19561	1638	861	777	67
68	37750	18966	18784	1705	895	810	68
69	36045	18071	17974	1767	926	841	69
70	34278	17145	17133	1825	954	871	70
71	32453	16191	16262	1876	978	898	71
72	30577	15213	15364	1921	999	922	72
73	28656	14214	14442	1955	1013	942	73
74	26701	13201	13500	1980	1022	958	74
75	24721	12179	12542	1991	1023	968	75
76	22730	11156	11574	1987	1016	971	76
77	20743	10140	10603	1966	1000	966	77
78	18777	9140	9637	1931	977	954	78
79	16846	8163	8683	1875	943	932	79
80	14971	7220	7751	1803	902	901	80
81	13168	6318	6850	1713	851	862	81
82	11455	5467	5988	1608	794	814	82
83	9847	4673	5174	1491	731	760	83
84	8356	3942	4414	1360	662	698	84
85	6996	3280	3716	1224	591	633	85
86	5772	2689	3083	1084	520	564	86
87	4688	2169	2519	943	448	495	87
88	3745	1721	2024	805	380	425	88
89	2940	1341	1599	675	316	359	89
90	2265	1025	1240	555	257	298	90
91	1710	768	942	444	204	240	91
92	1266	564	702	350	159	191	92
93	916	405	511	269	122	147	93
94	647	283	364	201	89	112	94
95	446	194	252	146	65	81	95
96	300	129	171	104	45	59	96
97	196	84	112	71	31	40	97
98	125	53	72	48	21	27	98
99	77	32	45	31	13	18	99
100	46	19	27	19	8	11	100
101	27	11	16	12	5	7	101
102	15	6	9	7	3	4	102
103	8	3	5	4	1	3	103
104	4	2	2	2	1	1	104
105	2	1	1	1	1	....	105
106	1	..	1	1	....	1	106

Table D.—HEALTHY DISTRICTS. Persons.

Age.	**Dying** in each year of age 0—1, 1—2, to 105—106.	**Born** and **living** at each age.	**Sum** of the **numbers born** and **living** at each age (x) from x to the last age in the Table.	**Population**, or the **living** in each year of age 0 to 1, 1 to 2, &c.	(1) **Sum** of the **living**, and of the **living** of every age (x) and upwards to the last age in the Table; also (2) the **years** which the persons (l_x) **will live**.	(1) The **years** which the persons at the age (x) and upwards **will live**; also (2) the **years** which they **have lived** over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x+l_{x+1})=l_{x+1}+\frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x+Q_{x+1})$ $=Y_{x+1}+(Q_{x+1}+\frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
0	10295	100000	4951908	92611	4899665	166209701	0
1	3005	89705	4851908	88202	4807054	161356341	1
2	1885	86700	4762203	85758	4718852	156593388	2
3	1305	84815	4675503	84162	4633094	151917415	3
4	1051	83510	4590688	82985	4548932	147326402	4
5	847	82459	4507178	82036	4465947	142818963	5
6	682	81612	4424719	81270	4383911	138394034	6
7	555	80930	4343107	80653	4302641	134050757	7
8	459	80375	4262177	80145	4221988	129788443	8
9	391	79916	4181802	79721	4141843	125606527	9
10	347	79525	4101886	79352	4062122	121504545	10
11	324	79178	4022361	79016	3982770	117482099	11
12	319	78854	3943183	78694	3903754	113538837	12
13	328	78535	3864329	78371	3825060	109674430	13
14	350	78207	3785794	78032	3746689	105888556	14
15	379	77857	3707587	77668	3668657	102180882	15
16	414	77478	3629730	77271	3590989	98551059	16
17	451	77064	3552252	76838	3513718	94998706	17
18	489	76613	3475188	76369	3436880	91523407	18
19	524	76124	3398575	75862	3360511	88124711	19
20	552	75600	3322451	75323	3284649	84802131	20
21	562	75048	3246851	74767	3209326	81555144	21
22	571	74486	3171803	74201	3134559	78383202	22
23	577	73915	3097317	73626	3060358	75285743	23
24	583	73338	3023402	73047	2986732	72262198	24
25	588	72755	2950064	72461	2913685	69311989	25
26	591	72167	2877309	71872	2841224	66434535	26
27	593	71576	2805142	71279	2769352	63629247	27
28	595	70983	2733566	70685	2698073	60895535	28
29	596	70388	2662583	70091	2627388	58232804	29
30	598	69792	2592195	69493	2557297	55640462	30
31	599	69194	2522403	68894	2487804	53117911	31
32	599	68595	2453209	68296	2418910	50664554	32
33	601	67996	2384614	67695	2350614	48279792	33
34	601	67395	2316618	67095	2282919	45963025	34
35	603	66794	2249223	66492	2215824	43713654	35
36	604	66191	2182429	65889	2149332	41531076	36
37	608	65587	2116238	65283	2083443	39414688	37
38	610	64979	2050651	64674	2018160	37363887	38
39	613	64369	1985672	64062	1953486	35378064	39
40	618	63756	1921303	63447	1889424	33456609	40
41	623	63138	1857547	62827	1825977	31598909	41
42	630	62515	1794409	62200	1763150	29804345	42
43	638	61885	1731894	61566	1700950	28072295	43
44	645	61247	1670009	60925	1639384	26402128	44
45	656	60602	1608762	60274	1578459	24793206	45
46	666	59946	1548160	59612	1518185	23244885	46
47	679	59280	1488214	58941	1458573	21756505	47
48	692	58601	1428934	58255	1399632	20327403	48
49	706	57909	1370333	57556	1341377	18956899	49

50	722	57203	1312424	56842	1283821	17644300	50
51	740	56481	1255221	56111	1226979	16388899	51
52	758	55741	1198740	55362	1170868	15189976	52
53	777	54983	1142999	54594	1115506	14046789	53
54	798	54206	1088016	53808	1060912	12958580	54
55	820	53408	1033810	52997	1007104	11924572	55
56	843	52588	980402	52167	954107	10943966	56
57	894	51745	927814	51298	901940	10015943	57
58	956	50851	876069	50373	850642	9139652	58
59	1040	49895	825218	49375	800269	8314197	59
60	1123	48855	775323	48293	750894	7538615	60
61	1205	47732	726468	47130	702601	6811867	61
62	1281	46527	678736	45887	655471	6132831	62
63	1356	45246	632209	44568	609584	5500304	63
64	1430	43890	586963	43175	565016	4913004	64
65	1501	42460	543073	41709	521841	4369575	65
66	1571	40959	500613	40173	480132	3868589	66
67	1638	39388	459654	38570	439959	3408544	67
68	1705	37750	420266	36897	401389	2987869	68
69	1767	36045	382516	35161	364492	2604929	69
70	1825	34278	346471	33366	329331	2258017	70
71	1876	32453	312193	31515	295965	1945369	71
72	1921	30577	279740	29617	264450	1665162	72
73	1955	28656	249163	27678	234833	1415520	73
74	1980	26701	220507	25711	207155	1194527	74
75	1991	24721	193806	23726	181444	1000227	75
76	1987	22730	169085	21736	157718	830646	76
77	1966	20743	146355	19760	135982	683796	77
78	1931	18777	125612	17811	116222	557694	78
79	1875	16846	106835	15909	98411	450377	79
80	1803	14971	89989	14070	82502	359921	80
81	1713	13168	75018	12311	68432	284454	81
82	1608	11455	61850	10651	56121	222178	82
83	1491	9847	50395	9102	45470	171382	83
84	1360	8356	40548	7676	36368	130463	84
85	1224	6996	32192	6383	28692	97933	85
86	1084	5772	25196	5230	22309	72432	86
87	943	4688	19424	4217	17079	52739	87
88	805	3745	14736	3342	12862	37768	88
89	675	2940	10991	2603	9520	26577	89
90	555	2265	8051	1988	6917	18358	90
91	444	1710	5786	1488	4929	12436	91
92	350	1266	4076	1090	3441	8251	92
93	269	916	2810	782	2351	5355	93
94	201	647	1894	547	1569	3395	94
95	146	446	1247	372	1022	2099	95
96	104	300	801	249	650	1263	96
97	71	196	501	160	401	737	97
98	48	125	305	101	241	416	98
99	31	77	180	61	140	226	99
100	19	46	103	37	79	116	100
101	12	27	57	21	42	56	101
102	7	15	30	11	21	24	102
103	4	8	15	7	10	9	103
104	2	4	7	2	3	3	104
105	1	2	3	1	1	...	105
106	1	1	1	...	...	...	106

TABLE E.—HEALTHY DISTRICTS. MALES.

Age.	**Dying** in each year of age 0—1, 1—2, to 104—105.	Born and **living** at each age.	**Sum** of the **numbers** born and **living** at each age (x) from x to the last age in the Table.	**Population**, or the **living** in each year of age 0 to 1, 1 to 2, &c.	(1) **Sum** of the **living**, and of the **living** of every age (x) and upwards to the last age in the Table; also (2) the **years** which the males (l_x) **will live.**	(1) The **years** which the males at the age (x) and upwards **will live**; also (2) the **years** which they **have lived** over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x+l_{x+1})=l_{x+1}+\frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x+Q_{x+1})=Y_{x+1}+(Q_{x+1}+\frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
0	5767	51125	2509635	46915*	2482745	84008921	0
1	1591	45358	2458510	44562	2435830	81549633	1
2	953	43767	2413152	43291	2391268	79136084	2
3	661	42814	2369385	42483	2347977	76766462	3
4	532	42153	2326571	41887	2305494	74439726	4
5	427	41621	2284418	41408	2263607	72155176	5
6	341	41194	2242797	41023	2222199	69912273	6
7	275	40853	2201603	40716	2181176	67710585	7
8	223	40578	2160750	40466	2140460	65549767	8
9	186	40355	2120172	40262	2099994	63429540	9
10	161	40169	2079817	40089	2059732	61349677	10
11	146	40008	2039648	39935	2019643	59309990	11
12	142	39862	1999640	39791	1979708	57310314	12
13	144	39720	1959778	39648	1939917	55350502	13
14	154	39576	1920058	39499	1900269	53430409	14
15	168	39422	1880482	39338	1860770	51549889	15
16	186	39254	1841060	39161	1821432	49708788	16
17	205	39068	1801806	38965	1782271	47906937	17
18	227	38863	1762738	38750	1743306	46144148	18
19	248	38636	1723875	38512	1704556	44420217	19
20	267	38388	1685239	38254	1666044	42734917	20
21	272	38121	1646851	37985	1627790	41088000	21
22	277	37849	1608730	37711	1589805	39479203	22
23	281	37572	1570881	37431	1552094	37908253	23
24	284	37291	1533309	37149	1514663	36374875	24
25	287	37007	1496018	36864	1477514	34878786	25
26	288	36720	1459011	36576	1440650	33419704	26
27	289	36432	1422291	36287	1404074	31997342	27
28	290	36143	1385859	35998	1367787	30611412	28
29	291	35853	1349716	35708	1331789	29261624	29
30	292	35562	1313863	35416	1296081	27947689	30
31	292	35270	1278301	35124	1260665	26669316	31
32	292	34978	1243031	34832	1225541	25426213	32
33	293	34686	1208053	34539	1190709	24218088	33
34	293	34393	1173367	34247	1156170	23044648	34
35	295	34100	1138974	33952	1121923	21905602	35
36	296	33805	1104874	33657	1087971	20800655	36
37	298	33509	1071069	33360	1054314	19729512	37
38	300	33211	1037560	33061	1020954	18691878	38
39	302	32911	1004349	32760	987893	17687455	39
40	306	32609	971438	32456	955133	16715942	40
41	310	32303	938829	32148	922677	15777037	41
42	315	31993	906526	31836	890529	14870434	42
43	320	31678	874533	31518	858693	13995823	43
44	326	31358	842855	31195	827175	13152889	44
45	334	31032	811497	30865	795980	12341311	45
46	341	30698	780465	30527	765115	11560764	46
47	350	30357	749767	30182	734588	10810912	47
48	360	30007	719410	29827	704406	10091415	48
49	370	29647	689403	29462	674579	9401923	49

50	381	29277	659756	29087	645117	8742075	50
51	394	28896	630479	28699	616030	8111501	51
52	407	28502	601583	28298	587331	7509821	52
53	420	28095	573081	27885	559033	6936639	53
54	435	27675	544986	27458	531148	6391548	54
55	451	27240	517311	27014	503690	5874129	55
56	467	26789	490071	26556	476676	5383946	56
57	483	26322	463282	26080	450120	4920548	57
58	501	25839	436960	25589	424040	4483468	58
59	542	25338	411121	25067	398451	4072223	59
60	587	24796	385783	24502	373384	3686305	60
61	631	24209	360987	23894	348882	3325172	61
62	672	23578	336778	23242	324988	2988237	62
63	712	22906	313200	22550	301746	2674870	63
64	752	22194	290294	21818	279196	2384399	64
65	789	21442	268100	21047	257378	2116112	65
66	826	20653	246658	20240	236331	1869258	66
67	861	19827	226005	19397	216091	1643047	67
68	895	18966	206178	18518	196694	1436654	68
69	926	18071	187212	17608	178176	1249219	69
70	954	17145	169141	16668	160568	1079847	70
71	978	16191	151996	15702	143900	927613	71
72	999	15213	135805	14714	128198	791564	72
73	1013	14214	120592	13707	113484	670723	73
74	1022	13201	106378	12690	99777	564093	74
75	1023	12179	93177	11668	87087	470661	75
76	1016	11156	80998	10648	75419	389408	76
77	1000	10140	69842	9640	64771	319313	77
78	977	9140	59702	8651	55131	259362	78
79	943	8163	50562	7692	46480	208556	79
80	902	7220	42399	6769	38788	165922	80
81	851	6318	35179	5892	32019	130519	81
82	794	5467	28861	5070	26127	101446	82
83	731	4673	23394	4308	21057	77854	83
84	662	3942	18721	3611	16749	58951	84
85	591	3280	14779	2984	13138	44007	85
86	520	2689	11499	2429	10154	32361	86
87	448	2169	8810	1945	7725	23422	87
88	380	1721	6641	1531	5780	16669	88
89	316	1341	4920	1183	4249	11655	89
90	257	1025	3579	897	3066	7997	90
91	204	768	2554	666	2169	5380	91
92	159	564	1786	484	1503	3544	92
93	122	405	1222	344	1019	2283	93
94	89	283	817	239	675	1436	94
95	65	194	534	161	436	880	95
96	45	129	340	107	275	525	96
97	31	84	211	68	168	303	97
98	21	53	127	43	100	169	98
99	13	32	74	25	57	91	99
100	8	19	42	15	32	46	100
101	5	11	23	9	17	22	101
102	3	6	12	4	8	9	102
103	1	3	6	3	4	3	103
104	1	2	3	1	1	1	104
105	1	1	1	...	...	...	105
106	...	...	...	...	...	...	106

* P_0 is $\frac{1}{2}(l_0+l_1)\times(\cdot9725)$. The factor ·9725 has been introduced, as the number living in the first year is less than the arithmetical mean of those born and surviving a year.

TABLE F.—HEALTHY DISTRICTS. FEMALES.

Age.	**Dying** in each year of age 0—1, 1—2, to 105—106.	**Born** and **living** at each age.	**Sum** of the **numbers born** and **living** at each age (x) from x to the last age in the Table.	**Population**, or the **living** in each year of age 0 to 1, 1 to 2, &c.	(1) **Sum** of the **living**, and of the **living** of every age (x) and upwards to the last age in the Table; also (2) the **years** which the females (l_x) **will live**.	(1) The **years** which the females at the age (x) and upwards **will live**; also (2) the **years** which they **have lived** over x.	Age.
		Σd_x.	Σl_x.	$\frac{1}{2}(l_x+l_{x+1})=l_{x+1}+\frac{1}{2}d_x$.	ΣP_x.	$\Sigma\frac{1}{2}(Q_x+Q_{x+1}) = Y_{x+1}+(Q_{x+1}+\frac{1}{2}P_x)$.	
x.	d_x.	l_x.	L_x.	P_x.	Q_x.	Y_x.	x.
0	4528	48875	2442273	45696†	2416920	82200780	0
1	1414	44347	2393398	43640	2371224	79806708	1
2	932	42933	2349051	42467	2327584	77457304	2
3	644	42001	2306118	41679	2285117	75150953	3
4	519	41357	2264117	41098	2243438	72886676	4
5	420	40838	2222760	40628	2202340	70663787	5
6	341	40418	2181922	40247	2161712	68481761	6
7	280	40077	2141504	39937	2121465	66340172	7
8	236	39797	2101427	39679	2081528	64238676	8
9	205	39561	2061630	39459	2041849	62176987	9
10	186	39356	2022069	39263	2002390	60154868	10
11	178	39170	1982713	39081	1963127	58172109	11
12	177	38992	1943543	38903	1924046	56228523	12
13	184	38815	1904551	38723	1885143	54323928	13
14	196	38631	1865736	38533	1846420	52458147	14
15	211	38435	1827105	38330	1807887	50630993	15
16	228	38224	1788670	38110	1769557	48842271	16
17	246	37996	1750446	37873	1731447	47091769	17
18	262	37750	1712450	37619	1693574	45379259	18
19	276	37488	1674700	37350	1655955	43704494	19
20	285	37212	1637212	37069	1618605	42067214	20
21	290	36927	1600000	36782	1581536	40467144	21
22	294	36637	1563073	36490	1544754	38903999	22
23	296	36343	1526436	36195	1508264	37377490	23
24	299	36047	1490093	35898	1472069	35887323	24
25	301	35748	1454046	35597	1436171	34433203	25
26	303	35447	1418298	35296	1400574	33014831	26
27	304	35144	1382851	34992	1365278	31631905	27
28	305	34840	1347707	34687	1330286	30284123	28
29	305	34535	1312867	34383	1295599	28971180	29
30	306	34230	1278332	34077	1261216	27692773	30
31	307	33924	1244102	33770	1227139	26448595	31
32	307	33617	1210178	33464	1193369	25238341	32
33	308	33310	1176561	33156	1159905	24061704	33
34	308	33002	1143251	32848	1126749	22918377	34
35	308	32694	1110249	32540	1093901	21808052	35
36	308	32386	1077555	32232	1061361	20730421	36
37	310	32078	1045169	31923	1029129	19685176	37
38	310	31768	1013091	31613	997206	18672009	38
39	311	31458	981323	31302	965593	17690609	39
40	312	31147	949865	30991	934291	16740667	40
41	313	30835	918718	30679	903300	15821872	41
42	315	30522	887883	30364	872621	14933911	42
43	318	30207	857361	30048	842257	14076472	43
44	319	29889	827154	29730	812209	13249239	44
45	322	29570	797265	29409	782479	12451895	45
46	325	29248	767695	29085	753070	11684121	46
47	329	28923	738447	28759	723985	10945593	47
48	332	28594	709524	28428	695226	10235988	48
49	336	28262	680930	28094	666798	9554976	49

50	341	27926	652668	27755	638704	8902225	50
51	346	27585	624742	27412	610949	8277398	51
52	351	27239	597157	27064	583537	7680155	52
53	357	26888	569918	26709	556473	7110150	53
54	363	26531	543030	26350	529764	6567032	54
55	369	26168	516499	25983	503414	6050443	55
56	376	25799	490331	25611	477431	5560020	56
57	411	25423	464532	25218	451820	5095395	57
58	455	25012	439109	24784	426602	4656184	58
59	498	24557	414097	24308	401818	4241974	59
60	536	24059	389540	23791	377510	3852310	60
61	574	23523	365481	23236	353719	3486695	61
62	609	22949	341958	22645	330483	3144594	62
63	644	22340	319009	22018	307838	2825434	63
64	678	21696	296669	21357	285820	2528605	64
65	712	21018	274973	20662	264463	2253463	65
66	745	20306	253955	19933	243801	1999331	66
67	777	19561	233649	19173	223868	1765497	67
68	810	18784	214088	18379	204695	1551215	68
69	841	17974	195304	17553	186316	1355710	69
70	871	17133	177330	16698	168763	1178170	70
71	898	16262	160197	15813	152065	1017756	71
72	922	15364	143935	14903	136252	873598	72
73	942	14442	128571	13971	121349	744797	73
74	958	13500	114129	13021	107378	630434	74
75	968	12542	100629	12058	94357	529566	75
76	971	11574	88087	11088	82299	441238	76
77	966	10603	76513	10120	71211	364483	77
78	954	9637	65910	9160	61091	298332	78
79	932	8683	56273	8217	51931	241821	79
80	901	7751	47590	7301	43714	193999	80
81	862	6850	39839	6419	36413	153935	81
82	814	5988	32989	5581	29994	120732	82
83	760	5174	27001	4794	24413	93528	83
84	698	4414	21827	4065	19619	71512	84
85	633	3716	17413	3399	15554	53926	85
86	564	3083	13697	2801	12155	40071	86
87	495	2519	10614	2272	9354	29317	87
88	425	2024	8095	1811	7082	21099	88
89	359	1599	6071	1420	5271	14922	89
90	298	1240	4472	1091	3851	10361	90
91	240	942	3232	822	2760	7056	91
92	191	702	2290	606	1938	4707	92
93	147	511	1588	438	1332	3072	93
94	112	364	1077	308	894	1959	94
95	81	252	713	211	586	1219	95
96	59	171	461	142	375	738	96
97	40	112	290	92	233	434	97
98	27	72	178	58	141	247	98
99	18	45	106	36	83	135	99
100	11	27	61	22	47	70	100
101	7	16	34	12	25	34	101
102	4	9	18	7	13	15	102
103	3	5	9	4	6	6	103
104	1	2	4	1	2	2	104
105	1*	1*	2	1	1	..	105
106	1*	1*	1	..	..	..	106

* The values of l_{104}, l_{105}, and l_{106}, decimally carried out, are 2·490, 1·250, and 0·603; and their differences are 1·240, 0·647, and 0·325. The apparent anomaly that no death happens between the ages 105 and 106, arises from the omission of decimals.

† P_0 is $\frac{1}{2}(l_0+l_1)\times(\cdot 98037)$. The factor ·98037 has been introduced, as the number living in the first year is less than the arithmetical mean of those born and surviving a year.

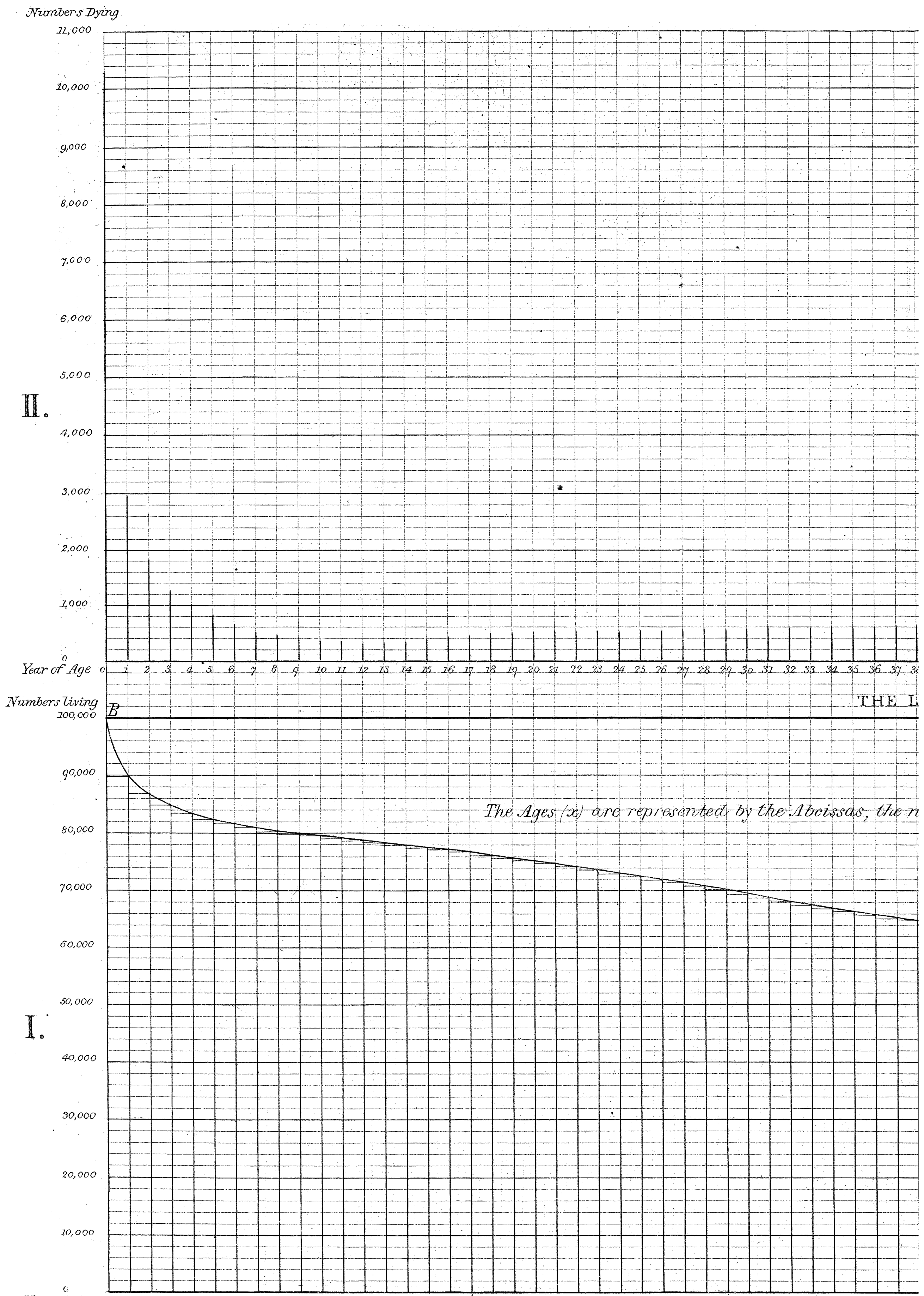
TABLE G.—HEALTHY DISTRICTS LIFE-TABLE.

The MEAN AFTER-LIFETIME (or the *Expectation of Life*) at the age x, and at the age x and upwards; also the MEAN AGES of the LIVING and the MEAN AGES AT DEATH. (Constructed from Tables D, E, F.)

PERSONS.					
Age (or past Lifetime).	Mean After-lifetime of Persons of the Age x.	Mean After-lifetime of Persons of the Age x and *upwards*.	Mean Age of Persons living of the Age x and *upwards*.	Mean Age at Death: Of Persons actually living at the Age x.	Mean Age at Death: Of Persons actually living at the Age x and *upwards*.
x.	$A_x=\frac{Q_x}{D_x}$.	$A'_x=\frac{Y_x}{Q_x}$.	$x+A'_x$.	$x+A_x$.	$x+2A'_x$.
0	49·00	33·92	33·92	49·00	67·84
5	54·16	31·98	36·98	59·16	68·96
10	51·08	29·91	39·91	61·08	69·82
15	47·12	27·85	42·85	62·12	70·70
20	43·45	25·82	45·82	63·45	71·64
25	40·05	23·79	48·79	65·05	72·58
30	36·64	21·76	51·76	66·64	73·52
35	33·17	19·73	54·73	68·17	74·46
40	29·64	17·71	57·71	69·64	75·42
45	26·05	15·71	60·71	71·05	76·42
50	22·44	13·74	63·74	72·44	77·48
55	18·86	11·84	66·84	73·86	78·68
60	15·37	10·04	70·04	75·37	80·08
65	12·29	8·37	73·37	77·29	81·74
70	9·61	6·86	76·86	79·61	83·72
75	7·34	5·51	80·51	82·34	86·02
80	5·51	4·36	84·36	85·51	88·72
85	4·10	3·41	88·41	89·10	91·82
90	3·05	2·65	92·65	93·05	95·30
95	2·29	2·05	97·05	97·29	99·10
100	1·72	1·47	101·47	101·72	102·94

Age (or past-Lifetime).	MALES.		FEMALES.	
	Mean After-lifetime of Males of the Age x.	Mean Age at Death of Males actually living at the Age x.	Mean After-lifetime of Females of the Age x.	Mean Age at Death of Females actually living at the Age x.
x.	$A_x=\frac{Q_x}{D_x}$.	$x+A_x$.	$A_x=\frac{Q_x}{D_x}$.	$x+A_x$.
0	48·56	48·56	49·45	49·45
5	54·39	59·39	53·93	58·93
10	51·28	61·28	50·88	60·88
15	47·20	62·20	47·04	62·04
20	43·40	63·40	43·50	63·50
25	39·93	64·93	40·18	65·18
30	36·45	66·45	36·85	66·85
35	32·90	67·90	33·46	68·46
40	29·29	69·29	30·00	70·00
45	25·65	70·65	26·46	71·46
50	22·03	72·03	22·87	72·87
55	18·49	73·49	19·24	74·24
60	15·06	75·06	15·69	75·69
65	12·00	77·00	12·58	77·58
70	9·37	79·37	9·85	79·85
75	7·15	82·15	7·52	82·52
80	5·37	85·37	5·64	85·64
85	4·01	89·01	4·19	89·19
90	2·99	92·99	3·11	93·11
95	2·25	97·25	2·32	97·32
100	1·69	101·69	1·75	101·75

The Table may be read thus:—Persons in the Healthy Districts of England of the precise age 20 will live on an average 43·45 years; while persons of the age of 20 and *upwards*, living in a normally constituted population of the same character, will live on an average 25·82 years. The mean age of persons of the age 20 and *upwards* is 45·82 years; the mean age at death of persons living at the precise age 20 will be 63·45, while the mean age at death of persons actually living at the age x and *upwards* will be 71·64 years.



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90,000
The Ages (x) are represented by the Abcissas, the n
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20,000
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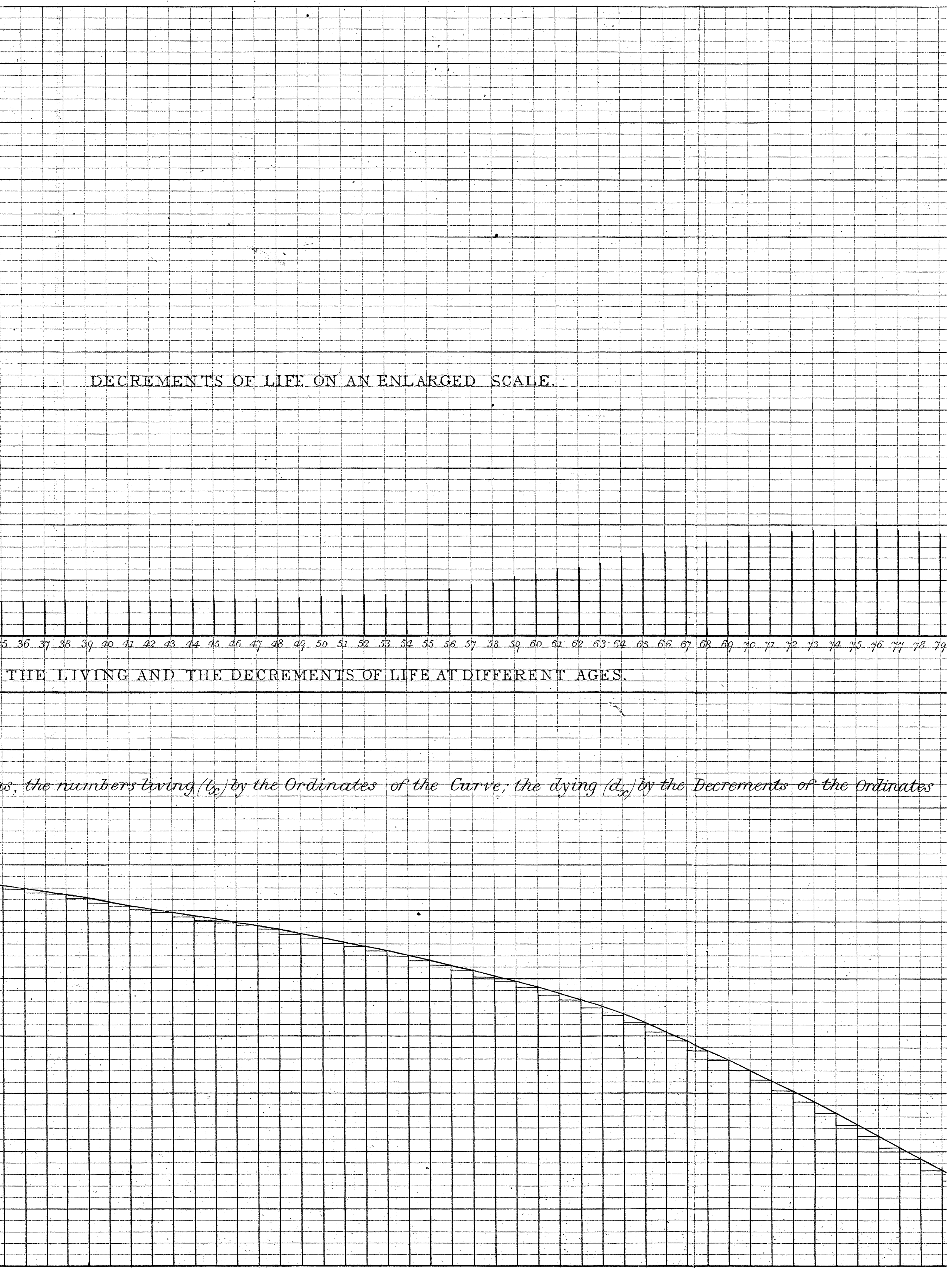
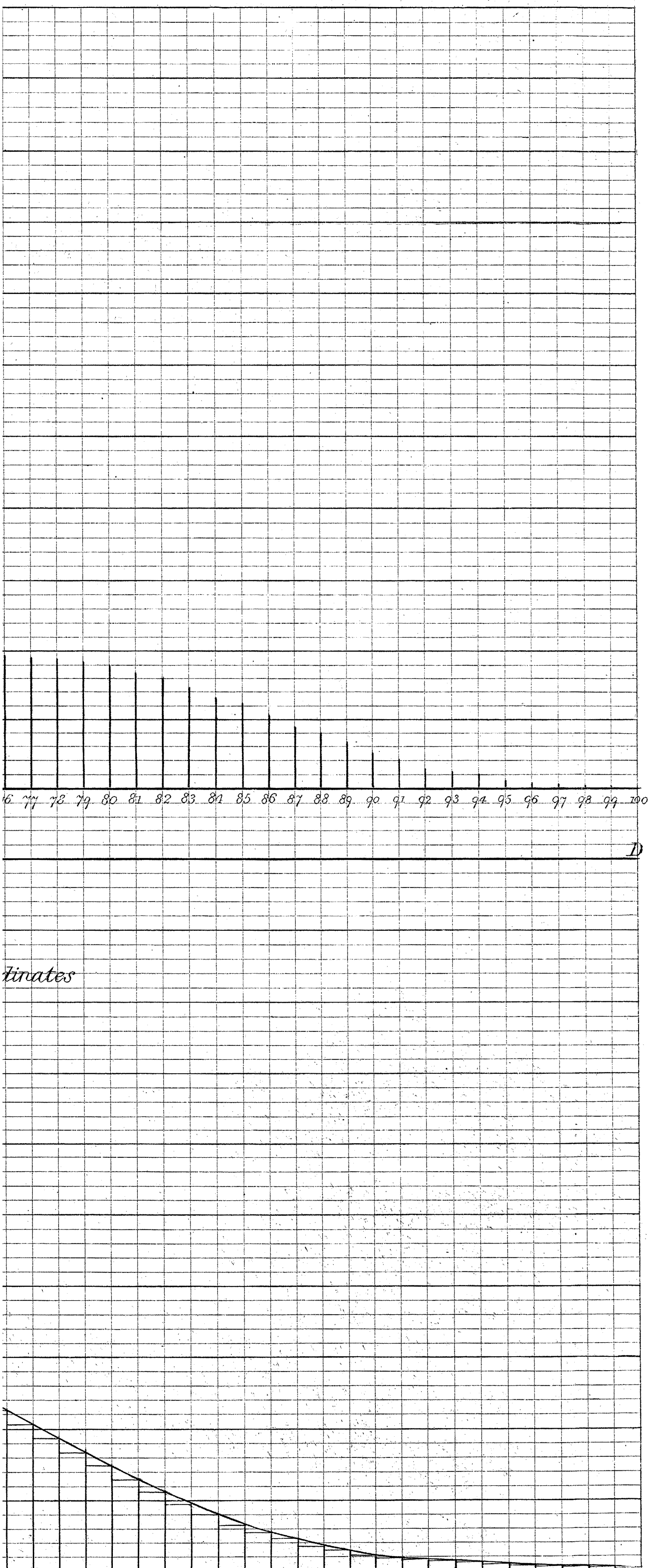
HEALTHY DISTRICTS.

LIFE TABLE DIAGRAMS.

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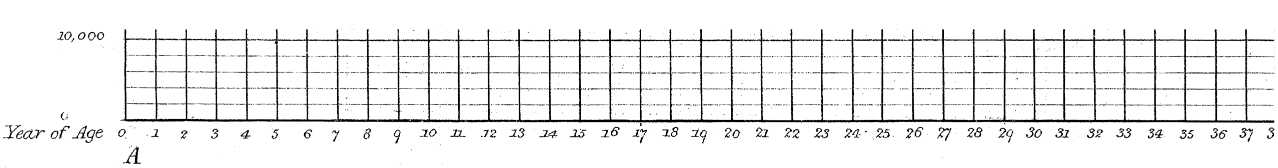
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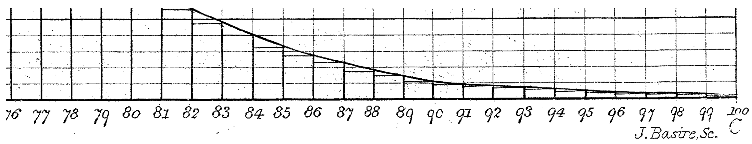
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C
J. Basire, Sc.

HEALTHY DISTRICTS.

LIFE TABLE DIAGRAMS.

Phil. Trans MDCCCLIX. Plate XLII.